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Research paper

# A finite element approach for modelling single-phase compressible flow in dual porosity systems

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ABSTRACT

Fluid flow in a rock formation that contains a network of fractures occurs through two coupled systems, the fracture network and the blocks of porous and permeable rock matrix. Dynamic modelling is challenging when low permeability of the rock matrix creates transient effects that can persist for long periods. In this paper, we describe a finite element (FE) based algorithm developed to model the flow of highly compressible gas through a fractured reservoir. The fracture network is represented by a two-dimensional (2D) FE mesh, coupled to a number of customised strings of one-dimensional (1D) elements representing individual matrix blocks. The use of multiple FEs to model each matrix block replaces the transfer function and facilitates accurate modelling of transient effects in the matrix blocks while still honouring the fully compressible nature of gas.

We use the algorithms to study some of the peculiarities of gas flow through fractured reservoirs with low matrix permeability. Matrix block geometry is shown to be an important parameter requiring accurate representation, particularly under transient flow conditions. We show how different geometric shapes can be reduced to 1D representations while still retaining much of the essential geometric information. It is shown that where transient effects occur, neglecting to capture them in the model can result in large errors. A conceptual system is suggested for classifying fractured reservoirs into six categories, each with a preferred modelling approach determined by the values of matrix permeability and fracture density. Finally, a case study is presented of a gas field producing from a low permeability fractured carbonate via two wells. A history match is successfully accomplished due to the flexible meshing capability of the finite element method and the transient modelling capabilities of the algorithms.



## 1. Introduction

The equations governing fluid flow through a fractured porous medium can be solved numerically by applying the 'dual porosity' model or some variation thereof. Two overlapping simulation grids (Aziz and Settari, 1979) represent the matrix and fracture networks respectively and are linked via transfer functions. This model is based on the premise that the fracture network provides the conduit through which reservoir fluid moves while the porous matrix material provides the storage space. Barenblatt (1960), Barenblatt and Zheltov (1960) and Barenblatt et al. (1960) presented a theory in support of dual porosity models and proposed the use of Darcy's equation to describe the macroscopic flow of fluids through the fracture network. They introduced a simple pseudo-steady state transfer function. Warren and Root (1963) developed further the dual porosity concept and derived formulae for the shape factors for simple geometries for semi-steady state flow. Kazemi et al. (1969) extended the dual porosity model to interpret interference test results. Bai et al. (1994) considered the 'dual porosity dual permeability' model in which fluid flow also occurs between adjacent matrix grid blocks. Lim and Aziz (1995) developed time-dependent analytical transfer functions for flow of slightly compressible fluid which accounts for transient effects.

Much work has been done to improve the transfer function. Pruess and Narasimham (1985a) studied numerical alternatives to the analytical transfer functions and championed the multiple interacting continua method (MINC) in which matrix blocks are discretised into a sequence of nested shells. The work presented in this paper has its origin in the MINC approach. Gilman (1986) presented results of work on the numerical modelling of single-phase compressible flow using finite differences and subdivided matrix blocks for enhanced representation of the transfer of fluids under transient conditions. Kazemi et al. (1976) developed a 3D dual porosity simulator for two-phase immiscible flow incorporating imbibition. Wu and Pruess (1988) emphasised the difficulties caused by transient effects when modelling the recovery of oil by water imbibition, such as occurs in very low permeability matrix or where matrix blocks are large. Pruess and

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Wu (1989) presented analytical transfer functions derived from their MINC formulation. They suggested the use of sub-domain models for problems involving high fluid viscosity, large matrix blocks, or low matrix block permeability. Kazemi et al. (1992) presented a variety of transfer functions for different recovery processes.

De Swaan and Raimirez-Villa (1993) developed a time-dependent equation (based on work done by Pollard, 1953) to describe the single-phase flow from a single matrix block. They derived geometric factors for a cube and for a slab. Zimmerman and Bodvarsson (1995) demonstrated that matrix block size and geometry were both important parameters. Beckner et al. (1991) subdivided rectangular representative matrix blocks into rings and layers, to enhance pressure and saturation gradient resolution. They showed large differences in cumulative oil production between forecasts made using either discretised or unrefined matrix block models. Lu and Connell (2007) developed a transfer function for multi-species gas flow incorporating adsorption behaviour.

Behbahani et al. (2006) modelled the counter-current imbibition process in which water imbibes from fractures into oil-saturated matrix blocks. They developed a transfer function that could be applied to 1D models to closely match 2D models. Rangel-German and Kovscek (2006) developed a time-dependent shape factor and transfer function to account for transient effects that occur in partially submerged matrix blocks.

Finite element methods (FEMs) are well established for the modelling of groundwater problems (Cheng, 1978). Within the petroleum industry, finite difference methods have been developed extensively and are in common use, while FEMs have been used for specialised applications. Duguid and Lee (1977) and Duguid and Abel (1974) applied FE techniques to fluid flow in naturally fractured rock formations.

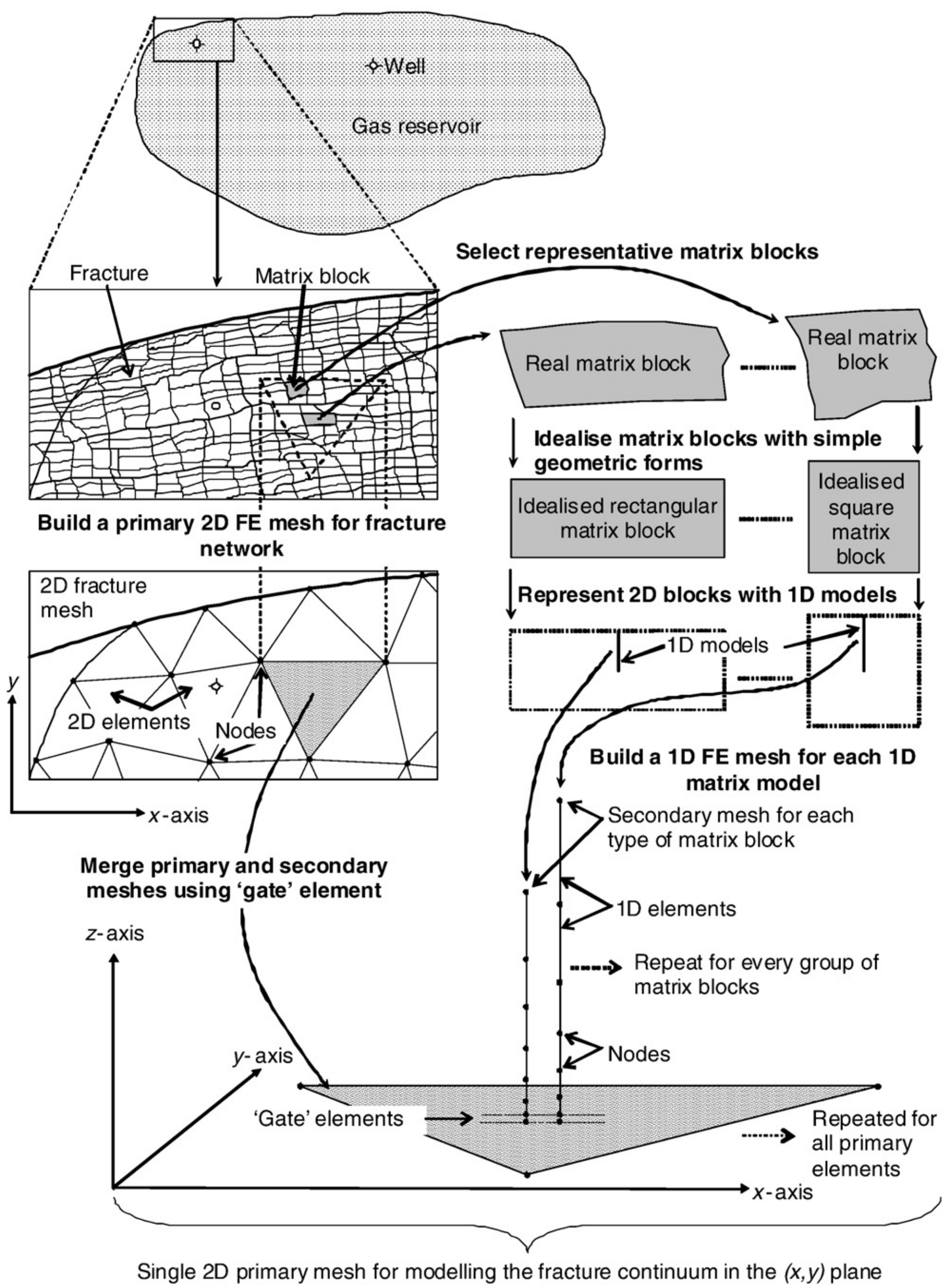


**Fig. 1.** Illustration of mesh definitions for the dual porosity model. The continuous component of the reservoir (the fracture medium) is modelled with a 2D primary mesh. The matrix blocks are modelled using a secondary set of 1D meshes coupled to the primary mesh.