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Research paper

An efficient method for smart well production optimisation

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ABSTRACT

A method for dynamic optimisation of water flooding with smart wells is presented. The method finds the optimal injection and production rates for every well segment of the smart wells. We formulate the problem as a constrained optimisation problem and state this problem as an augmented Lagrangian saddle point problem. Comparisons are made with a more traditional optimal control method based on solving the adjoint systems of equations. In the examples tested the method obtains same maximum profit while using less computational effort.

1. Introduction

As the oil resources of the world are becoming increasingly difficult to recover, it has become more important to produce existing fields as efficiently as possible and to decrease the development and operating costs. This problem is an optimisation problem where we want to maximise some profit function. In this paper we propose a method for maximising the net present value (NPV) of an oil reservoir, by reducing water production and increasing oil recovery at the same time as we are delaying water breakthrough. Optimal control theory methods have been used by many authors to solve this problem using the adjoint method, see e.g. (Asheim, 1988; Brouwer and Jansen, 2004; Lien et al., 2006; Pallav et al., 2005; Ramirez, 1987; Sudaryanto and Yortsos, 2001; Zakirov et al., 1996). However, in all these approaches it is required to solve the state equations exactly for each new estimate of the control variables, which is computationally expensive.

In this paper we formulate the optimisation problem as an augmented Lagrangian saddle point problem, and present a method for solving it by solving the Karush-Kuhn-Tucker (KKT) conditions for the augmented Lagrangian functional. By solving the KKT conditions sequentially, we avoid solving the state equations exactly for each new estimate of the control variables, thereby reducing the computational cost for finding a new estimate of the controls. Although the state equations are not fulfilled during the optimisation procedure, they will be so at convergence. Preliminary results have been presented earlier in (Doublet et al., 2006) and (Doublet et al., 2007).

The use of the augmented Lagrangian functional for constrained optimisation, with, was introduced by Hestenes in (Hestenes, 1969) and has been applied to optimal control problems later in (Ito and Kunisch, 1990; Kunisch and Tai, 1997). In this paper we make use of the augmented Lagrangian functional, but the solution approach is somewhat different than the one used in (Ito and Kunisch, 1990) and (Kunisch and Tai, 1997).

A proof of convergence for a quadratic optimisation problem with linear constraints is presented in the appendix, showing that it is possible to find a penalisation parameter that guarantees convergence in the quadratic case. The efficiency of the method is demonstrated through several numerical examples. We also present comparisons with the more traditional adjoint method.

2. Problem formulation

We consider a rectangular, heterogeneous, two-dimensional, two-phase (oil and water) reservoir with no-flow boundaries inspired by the model of Brouwer and Jansen (Brouwer and Jansen, 2004). The model is horizontal such that gravitational effects can be ignored. The capillary pressure is zero, and we use the Corey models to define the relative permeabilities. There is an injector along the left edge of the reservoir, and a producer along the right edge of the reservoir, as shown in Fig. 1. The two smart wells have several segments, indicated by black dots, such that the injection and the production can be controlled individually for each segment. Initially the reservoir is completely oil saturated, and at the start of operation water is injected in the segments on the left hand side of the reservoir, whilst we are producing from the segments at the right hand side of the reservoir. Initially, only oil is produced, but after a certain time we will start to produce both oil and water, and finally only water is produced. There is a profit associated with production of oil. However when producing both oil and water simultaneously we must remove the water from the oil, resulting in a cost associated with water production. The total production rate is fixed,

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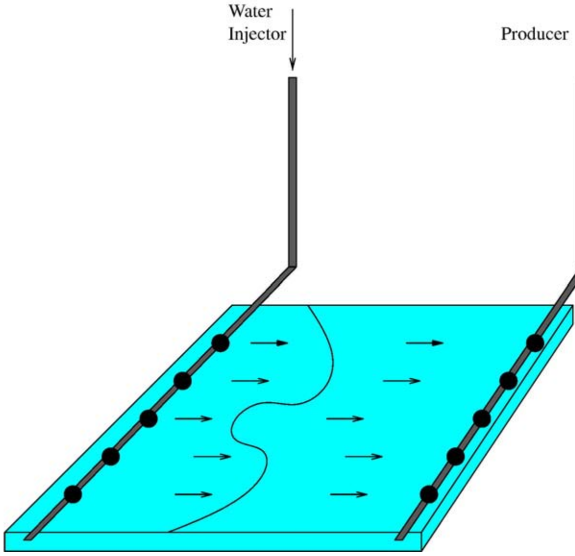

Fig. 1. Reservoir.

and we assume that we inject at the same rate as we produce. The profit may be controlled by adjusting the percentage of total injection/production in each of the segments, and our objective is to maximise the NPV from an oil reservoir by adjusting the individual segments injection and production rates at distinct times.

The reservoir flow equations are discretised using a standard cell centred grid with upstream weighting (Aziz and Settari, 1979) using backward Euler to approximate the time derivative. From the discretisation we derive a discrete time model of the two-phase conservation equations,

$$e^n(\hat{v}^n, \hat{u}^n, \hat{u}^{n-1}) = 0, \quad \text{for} \quad n = 1, \ldots N, \tag{1}$$

where $e^n-\{e_o^n, e_w^n\}^T$ is the residual column vector corresponding to the discretised version of the two-phase flow equations (Aziz and Settari, 1979), the superscript n denotes the discrete time step, N is the total number of time steps, $(\hat{u}^n)^T = \{p^{n^T}, S^{n^T}\}$ is a column vector consisting of the pressures and the water saturations in all the grid cells at time step n, and $\hat{v}^n$ is the column vector consisting of the control variables at time step n. The elements of $\hat{v}^n$ are related to the water injection and liquid production rates in the different segments of the wells.

Instead of using a well model, we will directly control water injection and liquid production at each segment such that the well segments are treated as source and sink terms. We assume that the total water injection rate equals the total liquid production rate and denote the total injection/production rate by Q.

Letting v_i^n denote the percent of total injection or production in segment i at time step n, we have the relations

$$\sum_{i=1}^{N_{inj}} v_i^n = 1, \quad \text{for} \quad n = 1, \ldots, N, \tag{2}$$

and

$$\sum_{i=1+N_{inj}}^{N_{prod}+N_{inj}} v_i^n = 1, \quad \text{for} \quad n = 1, \ldots, N, \tag{3}$$

where N_{inj} is the number of segments in the injector and N_{prod} is the number of segments in the producer. All the segments from number 1 to number N_{inj} are assumed to belong to the injection well, while the segments from number $1+N_{inj}$ to number $N_{prod}+N_{inj}$ are assumed to belong to the production well.

Moreover, the control variables must also satisfy the following inequality constraints, for $n=1,\ldots,N$,

$$0 \le v_i^n \le 1, \quad \text{for} \quad i = 1, \ldots, N_{inj} + N_{prod}. \tag{4}$$

Since only water is injected, the liquid rate equals the water rate for the segments in the injector. Thus we have that, for $n=1,\ldots,N$,

$$V_{wi}^n = v_i^n Q, \quad \text{for} \quad i = 1, \ldots, N_{inj}, \tag{5}$$

where V_{wi}^n is the water rate at injection segment i at time step n. In the segments of the producer, however, the liquid rate equals the sum of the water and oil rates so that, for $n=1,\ldots,N$,

$$V_{wi}^n + V_{oi}^n = -v_i^n Q, \quad \text{for} \quad i = 1 + N_{inj}, \ldots, N_{prod} + N_{inj}, \tag{6}$$

where V_{wi}^n and V_{oi}^n are the water and oil rates, respectively, at production segment i at time step n. The different phase production rates can be expressed as functions of the liquid rate and the fractional flow at the well segment so that

$$V_{wi}^n = -\frac{\lambda_{wi}^n}{\lambda_{wi}^n + \lambda_{oi}^n} v_i^n Q \quad \text{for} \quad i = 1 + N_{inj}, \ldots, N_{prod} + N_{inj}, \tag{7}$$

and

$$V_{oi}^n = -\frac{\lambda_{oi}^n}{\lambda_{oi}^n + \lambda_{wi}^n} v_i^n Q, \quad \text{for} \quad i = 1 + N_{inj}, \ldots, N_{prod} + N_{inj}, \tag{8}$$

where λ_{wi}^n and λ_{oi}^n are the water and oil mobilities, respectively, in the grid cell containing well segment i and at time step n.

2.1. Profit function

Our aim is to maximise net present value, by controlling the individual segment rates during the entire production period. The net present value (NPV), J, is given as

$$J(\hat{v}\ ,\hat{u}\ ) = \sum_{n=1}^{N} J^n(\hat{v}^n, \hat{u}^n), \tag{9}$$

with

$$J^n(\hat{v}^n, \hat{u}^n) = \Delta x \Delta y h \left[\sum_{i=1+N_{inj}}^{N_{prod}+N_{inj}} \frac{-I_w \cdot V_{wi}^n - I_o \cdot V_{oi}^n}{(1 + b/100)^{t^n}} \right] \Delta t^n, \tag{10}$$

where V_o and V_w are yearly produced volumes of water and oil, I_o and I_w are the revenue of oil and the cost of water produced per volume expressed in $/\mathrm{m}^3$, respectively, b is the annual interest rate expressed in %, Δx and Δy are the dimensions of the grid cells in, respectively, horizontal and vertical direction, Δt^n is the size of the nth time step, and $t^n = \Sigma_{i=1}^{n} \Delta t^i$ is the time expressed in years at time step n. Since the production rates of water and oil, V_{wi}^n and V_{oi}^n, are less than or equal to zero, I_o is a positive constant and I_w is a negative constant. The objective is to maximise the function (9) by adjusting the percentage of total injection and production in each of the individual segments of the injection well and the production well, respectively. If we at some point produce so much water that J^n in Eq. (10) is negative, then we set J^n to zero. This is done to maximise the profit over the field life