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Research paper

Modeling capillary pressure using capillary bundles with arbitrary cross-sections obtained from photomicrographs

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ABSTRACT

We describe a model for the distribution of two liquid phases in the void space of porous rock at capillary equilibrium. The model consists of a bundle of capillary tubes of arbitrary cross-sections, which are taken directly from photomicrographs of thin sections. In spite of its simplicity compared to a full 3D model, the proposed model retains some important features of realistic pore space geometry, in particular, curvature and roughness of the pore walls, which play an important role in the mechanism of wettability change.

Equilibrium configurations at primary drainage are simulated by the inscribed ball algorithm. An expression for drainage entry pressure is based on the balance of thermodynamic and mechanical energy. For a capillary tube of arbitrary cross-section, the equation reflecting this balance is solved numerically in contrast to analytical solutions published previously for the idealized polygonal shapes.

The model is used to compute drainage capillary pressure for a number of chalk sample images, and the results are compared to the experimentally measured mercury injection data. The observed discrepancies are believed to be caused mainly by the unrealistic accessibility of all the capillaries in the capillary bundle model, the resolution limit of the image, and the reduction of the real 3D geometry to 2D. The overestimated interconnectivity of the pores gives the main contribution to the observed discrepancy between the computed and the experimentally measured drainage capillary pressure.

A correction is proposed for the modeled capillary pressure curves based on the combined use of the information from the micro-images and the mercury injection data. Theoretically, the connectivity correction is justified by the representation of the porosity space and the invaded space as fractals. The simulated and measured data agree well provided the proposed corrections are accounted for.

1. Introduction

The stability of the wetting phase film covering the originally water-wet rock is influenced by the pore geometry, namely, by the curvature of the pore walls. The influence of the pore surface curvature on the wettability change was pointed out in (Salathiel, 1973). In a convex pore, like a circular cylinder or a sphere, the collapse of the wetting phase film is, obviously, more difficult than in a concave pore, it requires higher capillary pressure. Of course, it is difficult to imagine a strictly concave pore. A part of the wall surface is concave if the porous rock is made of convex grains, e.g. spheres or ellipsoids, and there is no cement or clay. The contacts of the grains can be convex if the rock is cemented, or have a "convex-concave" (i.e., saddle-type) curvature. These non-convex parts of the pore walls are more likely to be converted to oil-wet than the convex walls of the same pore (and their vicinities), and than the convex walls. An attempt to account for both factors, the disjoining pressure and the curvature, in interpretation of data for real reservoir rocks was reported by Skauge et al. (2004). From thin sections, it was observed that the rock, which revealed MWS ("mixed-wet small") behavior, had star-shaped pores due to absence of clay, while the rock which included clays had convex pores almost of spherical shape, and revealed MWL ("mixed-wet large") behavior. The sand grains constituting both rocks were mainly convex.

Obviously, to account for this curvature factor one has to have a realistic description of the pore space geometry, either in 2D, or preferably, in 3D. If the disjoining pressure is disregarded, the exact shape of equilibrium oil–water interface at a given constant capillary pressure reduces to a well known classical problem of a surface with constant mean curvature. One well known example of such surfaces from the every day life is soap films. In 2D geometry (like bundle of tubes of arbitrary shapes) this surface is easy to describe, it consists of segments of circular cylinders. This is due to the fact that one of the 2 principle curvatures is zero and thus the other one has to be constant. In 3D, the sum of the two principle curvatures has to be constant on the equilibrium interface; a relatively simple solution to this problem exists for surfaces of revolution (Delaunay, 1841). An

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example where this type of surfaces encounter is liquid bridges between two touching, or nearly touching, cylinders or spheres (Erle et al., 1971; Kralchevsky and Nagayama, 2001; Mayer and Stowe, 2005, 2006). We note in passing that according to the experimental observations (Graue et al., 2001) liquid bridges are responsible for multiphase liquid mass transport across fractures in fractured chalk. The hydraulic transmissibility of these bridges will, apparently, control the phase permeabilities across the fractures for a given fracture geometry.

From this brief review, we conclude that realistic pore space geometry is important in modeling of capillary pressure, both drainage and imbibition.

The obvious advantage of a full 3D model is difficult to fully realize, because: (1) creating a 3D geometrical model is difficult, and (2) modeling of the equilibrium oil–water interface cannot be accurately resolved in 3D for a sufficiently complex geometrical model.

Creating a 3D geometrical model of porous structure is difficult because of two main reasons. (1) To acquire high quality input data is difficult. Advanced techniques, like synchrotron CT (Silin et al., 2003), and a combined use of a focused ion beam to cut off a sub-micron thick layer of a rock and subsequent SEM imaging (Tomutsa and Silin, 2004; Tomutsa et al., 2007) are able to supply the 3D image of the pore space with sufficient resolution, i.e., the input information, which is further used for creating a 3D geometrical model. They however require synchrotron, which is not commonly available equipment. Because of this difficulty at least two alternative approaches exist which avoid 3D input data as discussed below. (2) Even with 3D input data available, geometrical modeling of an extremely complex pore structure is not straightforward (Lindquist, 2002).

Since the acquisition of 2D images is relatively simple and is achieved with high precision by electron microscope, the idea is to generate a realization of a homogeneous stochastic field, conditioned to the 2D images in one way or another. This is accomplished either by matching correlation functions derived from the 2D images, see the review in Refs. (Øren and Bakke, 2003; Wu et al., 2006), in particular, by matching the structural quantities which are the same in 2D and in 3D, namely, the two-point probability function and the lineal path function(Yeong and Torquato, 1998), or by using the Markov lattice as proposed in (Wu et al., 2006).

This approach is being pursued by P. E. Øren and his group and has been successfully utilized in a number of real cases see Refs. (Bakke and Øren, 1997; Kløv et al., 2003; Øren and Bakke, 2003). The input data are obtained from the analysis of thin sections, i.e., from 2D micro images of the rock in question. Then a 3D model is constructed by sequential modeling of sedimentation, compaction and diagenesis.

The problem of modeling of the equilibrium oil–water interface cannot be accurately resolved in 3D for a sufficiently complex geometrical model.

In the general case with a complex 3D geometry, the exact solution to this problem is unknown, and an effective numerical solution is not practically possible. The only practical way to solve the problem at hand is by introducing some “physically reasonable” approximation. An algorithm to analyze the pore space geometry in 3D, and one of the first, to the best of our knowledge, attempt to construct the oil–water interface in realistic pore space geometry at a given capillary pressure (i.e., at capillary equilibrium!) was presented by (Hazlett, 1995). The oil–water interface is constructed using the inscribed ball of a given radius. This radius is then assigned as a constant curvature radius to the whole surface. This is obviously not the exact solution (which is stated by (Tomutsa et al., 2007)), but it can be a reasonable approximation if the two principle radii of curvature are not very different, i.e., the surface (or the interface) is close to spherical. To make a judgment on the accuracy of this approximation in each particular case, it could be useful to have the curvature of the resulting surface evaluated.

On the contrary, modeling of the oil–water interface can be more accurately done for a 2D geometry. As already mentioned, the 2D images of a thin section can be easily acquired with a very high resolution using electron microscopy. This solves the problem with the input data and makes the 2D modeling a temptation. An obvious disadvantage in this case is that the 2D description of the pore space is unavoidably deficient and incomplete. To overcome this deficiency the analysis of 2D images has to be done in combination with alternative sources of information, such as e.g. mercury injection experiments. In this case the information on the characteristic pore shapes, including curvatures and sizes, will be gained from the photomicrographs, while the information on spatial interconnectivity of the pores missing in the 2D images will be (at least partly) gained from the fluid flow experiments.

The idea to use mercury injection for the pore-space characterization and derivation of the properties of the rocks at higher scales has been utilized in a large number of publications, in particular, to describe the pore space as fractals, see Refs. (Shen et al., 1995; Li, 2004a,b; Laroche and Vizika, 2005). An analysis of 2D images to interpret the results of mercury injection tests was presented by (Angulo et al., 1992). The results show that at high enough drainage pressure the capillary pressure curve reveals power-law behavior. The analysis of the fragmented 2D images seems to support the assumption that the pore space is fractal.

2. Modeling of primary drainage

In order to simulate the capillary pressure a 2D image has to be replaced by a corresponding 3D structure in one or another way. We select the bundle of (non-circular) tubes representation, each of the tubes is a cylinder with the cross-section taken from the image at hand. In this case the equilibrium configuration corresponding to a constant capillary pressure consists of: (1) oil–water interface represented by a segment of a circular cylinder, and (2) oil–solid interface. As already mentioned, this is the exact solution for the bundle of tubes model. The equilibrium configuration is computed using the inscribed ball algorithm (Silin et al., 2003). In reality, a water film exists between oil and solid, which is assumed sufficiently thin; its volume is neglected in the computations of saturation.

2.1. Entry pressure at primary drainage

In this section we describe an algorithm to numerically compute the entry pressure for a cylindrical pore of arbitrary cross-section which was implemented in the in-house software.

For a cylindrical tube of an arbitrary cross-section the drainage entry pressure is obtained from the analysis of virtual energy (Ma et al., 1996). For oil front advancing in a cylinder of arbitrary cross-section, the work is expressed in two equivalent forms giving the following equation:

$$P_{cow}A_o dx = \sigma_{ow}L_{ow}dx + \sigma_{os}L_{os}dx - \sigma_{ws}L_{os}dx \tag{1}$$

where A_o is the cross-sectional area occupied by oil, and L_{os} (L_{ow}) is the length of the interface between oil and solid (oil and water).

Taking into account the Young equation and Eq. (1) we easily obtain

$$\frac{P_{cow}}{\sigma_{ow}} = \frac{L_{ow} + L_{os}\cos\theta}{A_o}, (\sigma_{os} - \sigma_{ws} = \sigma_{ow}\cos\theta) \tag{2}$$

The mean curvature $C = \frac{P_{cow}}{\sigma_{ow}}$ has to be constant at any point on the oil–water interface. Away from the displacement front (at $x = -\infty$)