Journal of Petroleum Science and Engineering 67 (2009) 91–96

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Influence of the wicking process on the heat transfer in a homogeneous porous medium

J.P. Escandón [a], O. Bautista [a], F. Méndez [b,*], E. Bautista [a]

[a] SEPI, ESIME Azcapotzalco IPN, México, D. F., 02550, Mexico
[b] Facultad de Ingeniería, UNAM, México, D. F., 04510, Mexico

ARTICLE INFO

Article history:
Received 27 March 2008
Accepted 4 May 2009

Keywords:
wicking process
porous medium
convective heat transfer

ABSTRACT

In this work, we analyze the spontaneous wicking process of a fluid in a homogeneous porous medium taking into account that the medium is subject to the presence of a temperature gradient, including the gravity effects. We assume that the porous medium is found initially at temperature T_0 and pressure P_0; suddenly the lower part of the porous medium touches a liquid reservoir with temperature T_1 and pressure P_0 and begins the spontaneous wicking process into the porous medium. The physical influence of two nondimensional parameters such as the ratio of the characteristic thermal time to the characteristic wicking time, β and α defined as the ratio of the hydrostatic head of the imbibed fluid to the characteristic pressure difference between the wicking front and the dry zone of the porous medium, serves us to evaluate the position and velocity of the wicking front as well as the temperature profiles and the corresponding Nusselt numbers in the wetting zone. In particular, for small values of time we recover the well-known Washburn law. The numerical predictions show that the wicking and the temperature profiles depend on the above nondimensional parameters, revealing a clear deviation of the simple Washburn law.

1. Introduction

Transport processes through porous media play an important role in diverse applications, such as in geothermal operations, oil exploration, thermal insulation, design of solid-matrix heat exchangers, chemical catalytic reactors, and many others. The study of convective heat transfer and fluid flow in porous media has received great attention in recent years. Most of the earlier studies (Cheng and Minkowycz, 1977; Minkowycz and Cheng, 1987; Badr and Pop, 1988; Nakayama and Koyama, 1995; Sánchez et al., 2004) are based on Darcy's law neglecting the gravity effect, which states that the volume-averaged velocity is proportional to the pressure gradient. In order to model a more realistic situation it is necessary to include the gravity effects in the analysis of the heat convection in a porous medium. In general, we can identify the wicking as a spontaneous process for which a wetting viscous fluid displaces or pushes aside a less no-wetting viscous one. Sometimes, this action can only be sustained by capillary forces, without external pressure. In this last case, we treat spontaneous wicking. In the above discussion, we accept obviously that both fluids are immiscible. Therefore, in this class of theoretical and experimental studies the front of displacement of the interface is not known in advance and should be estimated as part of the problem. From the pioneer work of Washburn (1921) to predict the well-known displacement law, $h \sim t^{1/2}$, nowadays the specialized literature has been extended widely in order to take into account the influence of different factors. Recently, Alava et al. (2004) presented the state of the art in disordered media, emphasizing those aspects that have a strong influence on the interfacial description, following a statistical physics approach. Taking into account the random characteristics of disordered porous media, Alava et al. (2004) discussed the main features of wicking: rough interfaces between both phases, fluctuations of the fluid flow, quenched noise and nonlocal effects originated from a random environment and capillary forces. Therefore, there are many possible deviations from the simple Lucas–Washburn description and these new fundamental studies serve to understand better a great variety of technological applications. Nevertheless, there is another class of wicking processes, which under different physical conditions are only regulated by transport phenomena. Recognizing that the literature is scarce in this direction, Alava et al. (2004) suggested that additional efforts are required to understand simultaneously the wicking and transport effects.

Since the practical applications related to the analysis of simultaneous wicking-transport processes are very vast, here we have focused on the theoretical study of the heat transfer and wicking in a Darcian porous medium, including the gravity effect. In spite of the practical importance of this class of processes, few works have appeared in the past to understand such combined effects. Phillips (1991) noted that the presence of approximately uniform geothermal gradients in natural reservoirs can seriously affect the prediction of fluid dynamics. In fact, the work of Babadagli (1996, 2002) explores

* Corresponding author. Tel.: +52 55 56 22 81 03; fax: +52 55 56 22 81 06.
E-mail address: fmendez@servidor.unam.mx (F. Méndez).

0920-4105/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.petrol.2009.05.001

the influence of a temperature field on a reservoir of oil, showing that oil production by wicking can be controlled with the aid of a heat transfer process, injecting hot vapors. Under this situation, temperature gradients in the wetting and no-wetting phases during the wicking process are presented.

In an effort to understand the influence of wicking on the above heat transfer process, in the present work we deal theoretically the non-isothermal wicking of a liquid-saturated layer into a dry zone. The resulting governing equations are solved numerically with a conventional finite-difference scheme. The theoretical analysis is basically organized as follows: we present a motion equation to describe the temporal evolution of the wicking front. In contrast with other theories, the above equation is coupled to the energy equation for the wetting phase. The above formulation includes as a particular case, the well-known Lucas–Washburn law (Washburn, 1921), valid for isothermal wicking. For simplicity, we assume one dimensional flow. Afterward, we derive with the aid of an order of magnitude analysis the nondimensional governing equations together with the corresponding initial and boundary conditions. After some transformations, the governing equations are reduced to one equation which has been numerically solved by using the Crank–Nicolson numerical scheme. In particular, the numerical solutions show that the transient response of the wicking front is sensibly controlled by the parameter β.

2. Order of magnitude analysis and theoretical model

The physical model under study is shown in Fig. 1. A slender piece or sheet that consists of an air-saturated porous medium (with porosity ϕ) is found initially at temperature and pressure T_0 and P_0, respectively. We assume thermal equilibrium between the saturated air and the porous matrix. Suddenly, the lower part of the sheet touches a liquid reservoir at temperature T_1 and the same pressure P_0, causing a non-isothermal wicking process of the liquid into the porous medium. In addition, the wicking front $h=h(t)$ is characterized by a uniform capillary pressure, P_c. The origin of coordinates is located at the base of the sheet. We adopt one dimensional formulation; therefore, is enough to introduce a longitudinal or vertical coordinate y, which is measured upward in the direction of the wicking front.

The competition between thermal and dynamics penetrations generates a non-isothermal capillary flow, which is developed inside the porous medium. After an elapsed time t, the non-isothermal wicking front reaches an average distance, $h(t)$. Here, the meaning of an average distance $h(t)$ is to accept that the microscopic effects are neglected, in a first approximation (Alava et al., 2004). Therefore, the thermal and wicking effects introduce two times scales: the thermal penetration scale, t_{th}, and the wicking scale, t_w. An order of magnitude analysis permits to identify both scales. Thus, from the energy equation for the porous medium given by (Vafai, 2005):

$$(\rho c)_e \frac{\partial T}{\partial t} + (\rho c)_f U_D \frac{\partial T}{\partial y} = k_e \frac{\partial^2 T}{\partial y^2} \quad (1)$$

where $(\rho c)_e$ represent the effective heat capacity of the liquid-porous matrix system and is defined as $(\rho c)_e = \phi(\rho c)_f + (1-\phi)(\rho c)_s$. $\rho_{f(s)}$ and $c_{f(s)}$ are the density and specific heat of the wetting liquid (or porous medium). Here, the subscripts f and s represent wetting and porous-matrix conditions. U_D is the Darcy velocity of the fluid in the porous medium and k_e is the effective thermal conductivity. We assume that the wicking front temperature is found at thermal equilibrium with the dry zone.

An energy balance between the transported thermal energy by the motion of the liquid and the accumulation energy term dictates that,

$$\frac{(\rho c)_f (T_1 - T_0) U_{CD}}{h_{th}} \sim \frac{(\rho c)_e (T_1 - T_0)}{t_{th_{conv}}} \quad (2)$$

where U_{CD}, h_{th} and $t_{th_{conv}}$ represent the characteristic Darcy volume-average velocity associated with the velocity of the wicking front, the characteristic thermal height and the characteristic convective time scale, respectively.

From Eq. (2), the characteristic convective time scale is given as,

$$t_{th_{conv}} \sim \frac{(\rho c)_e h_{th}}{(\rho c)_f U_{CD}}. \quad (3)$$

In a similar way, from Eq. (1), a dominant balance between diffusive and accumulation terms, dictates that,

$$\frac{k_e (T_1 - T_0)}{h_{th}^2} \sim \frac{(\rho c)_e (T_1 - T_0)}{t_{th_{diff}}}; \quad (4)$$

using the above relationship, the characteristic diffusive time scale is given by,

$$t_{th_{diff}} \sim \frac{(\rho c)_e h_{th}^2}{k_e}. \quad (5)$$

In order to obtain the order of magnitude of the characteristic thermal height, we compare the diffusive and convective terms of the energy equation,

$$\frac{(\rho c)_f (T_1 - T_0) U_{CD}}{h_{th}} \sim \frac{k_e (T_1 - T_0)}{h_{th}^2}, \quad (6)$$

Obtaining that

$$h_{th} \sim \frac{k_e}{(\rho c)_f U_{CD}}. \quad (7)$$

The wicking volume-average velocity U_{CD} is easily derived from the driven capillary pressure gradient,

$$U_{CD} \sim \frac{K(P_0 - P_c)}{\mu h_w}, \quad (8)$$

where K, μ and h_w are the permeability of the porous medium, the liquid viscosity and the equilibrium height of the wicking front, respectively. This last characteristic length h_w is the equilibrium

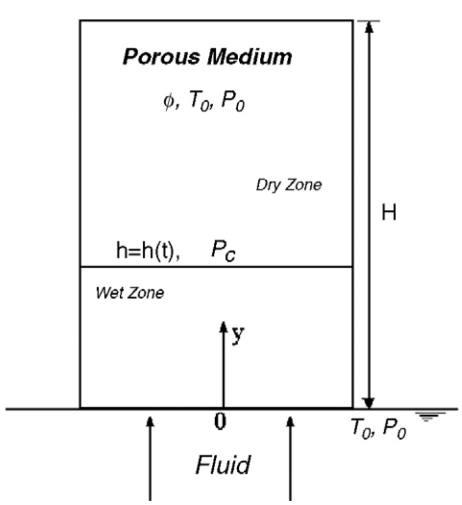

Fig. 1. Schematic view of the physical model during the non-isothermal wicking process in a porous medium.