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A simplified analytical method for estimating the productivity of a horizontal well producing at constant rate or constant pressure

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ABSTRACT

In this work, we present an analytical method for estimating the productivity of an infinite-conductivity horizontal well in a closed, box-shaped reservoir producing at either a constant rate or a constant pressure. It is based on approximate analytical solutions for the 2D inflow performance of an infinite-conductivity fracture in a rectangular drainage area and the near-well pressure drop for flow to a circular well in a vertical cross-section of the reservoir. The method can accommodate reservoirs with arbitrary horizontal and vertical aspect ratios, off-center wells, permeability anisotropy and skin factors. Being simple and straightforward, the method can be readily implemented in a standard spreadsheet program. Well productivities for depletion at constant rate are always higher than at constant pressure. The difference can be as much as 22% for a fully penetrating well in a thin, isotropic reservoir.

1. Introduction

The existing analytical methods for evaluating the productivity of horizontal wells in closed reservoirs are all based on production at a constant well rate [e.g. Babu and Odeh (1989), Goode and Kuchuk (1991)]. In practice, reservoir depletion very often occurs at a constant well pressure rather than at a constant rate, notably in the development of gas reservoirs. For a long time the conventional wisdom in petroleum engineering has been that well productivities are independent of the depletion mode and thus that a 'constant-rate' productivity also applies to a well producing at a constant pressure. A case in point is the productivity of an openhole well in the centre of a closed reservoir with a circular drainage area. For this flow configuration, Ehlig-Economides and Ramey (1981) showed mathematically that the constant-pressure productivity is identical to the constant-pressure productivity.

In the late nineties, Helmy and Wattenbarger (1998a) pointed out that well productivity in non-circular flow configurations does depend on depletion mode, particularly in flow configurations with a more linear flow pattern. This observation is in line with the analytical long-time solutions for the analogous linear heat flow problems discussed by Carlslaw and Jaeger (1959). From these solutions it appears that the linear productivity at constant-rate production is a factor $12/\pi^2$ (=1.22) larger than the one at constant-pressure production. The difference can be attributed to the different shapes of the pressure profile for the same production rate: the constant-rate profile takes the form of a quadratic relationship whereas the constant-pressure profile is a cosine function. This leads to different average pressures for the same production rate and thus to different productivities.

The depletion mode may have a significant bearing on the productivity of a horizontal well, for the flow pattern of horizontal wells generally consists of significant portions that exhibit linear flow, notably in the horizontal plane. Therefore, the constant-rate productivities predicted by the current analytical methods might be too optimistic for horizontal wells producing at a constant pressure.

To estimate the productivity of horizontal wells producing at both constant rate and constant pressure, Helmy and Wattenbarger (1998b) developed relatively simple productivity correlations based on a large number of numerical reservoir simulations. The reservoir model consisted of an isotropic, homogeneous, closed reservoir with the shape of a rectangular box and an openhole horizontal well parallel to one of the sides of the box. The correlations cover horizontal aspect ratios from 1 to 5, vertical aspect ratios from 2 to 100, off-center wells and well lengths of more than 0.2 times the longest side of the reservoir. Formation anisotropy can be taken into account by using equivalent reservoir and well parameters.

In this work, we present an alternative, analytical method for evaluating the productivity of horizontal wells in box-shaped reservoirs. It is based on the approach originally proposed by Borisov (1964), in which the essentially 3D flow problem is split into two simpler 2D problems: horizontal flow to a fracture and vertical flow to a circular well. The effect of the depletion mode can be incorporated through a simple modification of the constant-rate solutions for the horizontal flow problem. The method can accommodate reservoirs with arbitrary horizontal and vertical aspect ratios, off-center wells, permeability anisotropy and skin factors. Because of its simplicity, the method can be readily implemented in a standard spreadsheet program. Example calculations are presented

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that illustrate the effect of depletion mode and show how the method compares with previous analytical and numerical results.

The prime application of the method is as a quick-and-easy evaluation tool in conceptual development and scoping studies where horizontal wells are being considered. The method may also serve as a benchmark in numerical reservoir simulation studies.

2. Outline of the method

Fig. 1 depicts schematically the horizontal well configuration that is described by the simplified analytical method: a rectangular box-shaped reservoir with a horizontal well parallel to the long side of the box. The box has a length L, width w and thickness h. The well is an openhole well with a total length of L_w. Its position in the reservoir is defined by x_w, y_w and z_w, the coordinates of the well midpoint in the Cartesian coordinate system that is aligned parallel to the sides of the reservoir with its origin in the bottom-left corner point. In this coordinate system the horizontal well runs parallel to the x-axis. The reservoir is homogeneous and isotropic and contains a compressible fluid with a constant viscosity and constant compressibility. The outer boundaries of the reservoir are no-flow boundaries. The well is assumed to have an infinite conductivity, which implies a uniform pressure in the wellbore. The well is produced at either a constant rate or a constant pressure.

Our prime interest is the productivity at long production times, that is, when the pressure distribution and flow pattern in the reservoir are dominated by the outer boundaries of the reservoir. At long times, the ratio of production rate and pressure drawdown, i.e. the difference between average reservoir pressure and well pressure, approaches a constant value for both production at a constant rate and at a constant pressure. This ratio is a measure for the productivity of a well and is commonly called productivity index. The productivity index depends on the characteristics of the well, reservoir and reservoir fluid.

Following Borisov (1964), we can find the pressure drawdown for flow to a horizontal well by splitting the essentially 3D flow problem into two simpler 2D flow problems: flow to the well in a horizontal cross-section of the reservoir through the well and flow to the well in a vertical cross-section of the reservoir perpendicular to the well. See Fig. 2. The horizontal flow problem deals with the overall flow to the horizontal well excluding the converging vertical flow in the region near the wellbore. The latter is described by the vertical flow problem.

Let Δp_{xy} denote the pressure drawdown for flow in the horizontal plane and Δp_{wyz} the near-well vertical pressure drop. Then, according to Borisov, the 3D pressure drawdown Δp_{xyz} follows from

$$\Delta p_{xyz} = \overline{p}_{xyz} - p_w = \overline{p}_{xy} + \Delta p_{wyz} - p_w = \Delta p_{xy} + \Delta p_{wyz}. \quad (1)$$

Once the 3D pressure drawdown is known, the productivity index follows immediately.

In the following sections we shall derive approximate analytical relationships for both Δp_{xy} and Δp_{wyz}. These derivations stem largely from the analytical solution for the inflow performance of a fully penetrating, vertical well in a closed, rectangular box-shaped reservoir. Therefore, we shall first discuss this solution for production at both a constant rate and at a constant pressure.

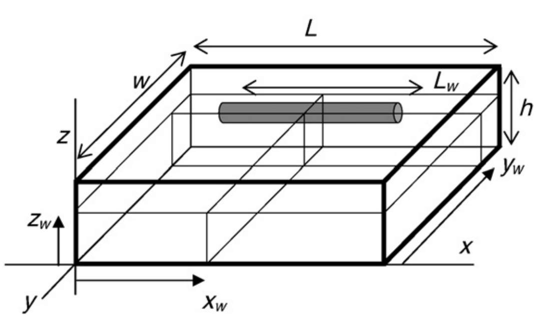

Fig. 1. Horizontal well in box-shaped reservoir.

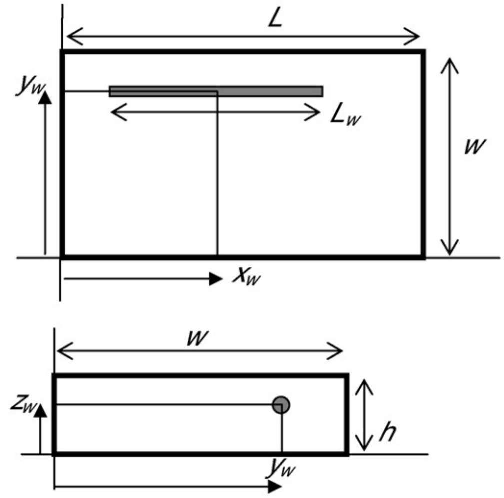

Fig. 2. Horizontal and vertical flow configurations.

3. Fully penetrating well in rectangular drainage area

Traditionally, the inflow performance relationship of a fully penetrating, vertical well in an arbitrarily shaped, plane reservoir is expressed in the same form as the radial inflow formula, viz.

$$\Delta p = \overline{p} - p_w = \frac{\mu q}{2\pi kh} \ln \frac{4A}{\gamma C_A r_w^2}, \quad (2)$$

where q is the production rate, μ the fluid viscosity, k the permeability, A the surface of the drainage area, γ the exponential of the Euler constant (=1.781…), C_A the Dietz shape factor [Dietz (1965)] and r_w the well radius. Eq. (2) applies to both constant-rate and constant-pressure depletion but then the Dietz shape factor is different. For a list of shape factors for various shapes of drainage areas for constant-rate production, see Earlougher (1977). Helmy and Wattenbarger (1998a) presented shape factors for production at constant pressure.

In the special case of rectangular drainage areas, one can derive the following closed-form expression for the pressure drawdown for constant-rate depletion at long times (seeAppendix A)

$$\Delta p_q = \overline{p}_q - p_w = \frac{\mu q}{2\pi kh} \Big\{ -\ln\left[2\pi r_{wD} \sin(\pi y_{wD})\right] + \frac{2\pi w}{3L}(L/w)\left[x_{wD}^3 + (1-x_{wD})^3\right] + F_A(x_{wD}, y_{wD}, L/w) \Big\}. \quad (3)$$

where the subscript q refers to constant-rate production, $r_{wD}=r_w/w$, $x_{wD}=x_w/L$, $y_{wD}=y_w/w$, and F_A is a function of well position and aspect ratio, defined inAppendix A. Eq. (3) holds good for well radii that are negligibly small with respect to the width of the reservoir ($r_{wD}<<1$) and for horizontal aspect ratios equal to or larger than unity ($L/w \geq 1$). The restriction to aspect ratios larger than unity presents no serious problem: aspect ratios smaller than unity can be handled by a 90-degrees rotation of the drainage area.

Equating the horizontal pressure drawdowns given by Eq. (2) and by Eq. (3), we obtain the following relationship for the shape factor

$$0.5 \ln\left[4/(\gamma C_{Aq})\right] = -\ln\left[2\pi \sin(\pi y_{wD})\right] - 0.5 \ln(w/L) + F_A(x_{wD}, y_{wD}, L/w) + \frac{2\pi w}{3L}\left[x_{wD}^3 + (1-x_{wD})^3\right]. \quad (4)$$