Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Petroleum Science and Engineering 51 (2006) 68–84
Journal of Petroleum Science & Engineering
www.elsevier.com/locate/petrol

Correction of basic equations for deep bed filtration with dispersion

J.E. Altoé F. [a], P. Bedrikovetsky [a,*], A.G. Siqueira [b], A.L.S. de Souza [b], F.S. Shecaira [b]

[a] *Department of Petroleum Exploration and Production Engineering, North Fluminense State University Lenep/UENF, Rod. Amaral Peixoto, km 163 – Av. Brenand, s/n° Imboacica – Macaé, RJ 27.925-310, Brazil*
[b] *Petrobras/Cenpes, Cidade Universitaria Q.7, Ilha Do Fundão, 21949-900 – Rio De Janeiro — RJ, Brazil*

Accepted 4 November 2005

Abstract

Deep bed filtration of particle suspensions in porous media occurs during water injection into oil reservoirs, drilling fluid invasion into reservoir productive zones, fines migration in oil fields, bacteria, virus or contaminant transport in groundwater, industrial filtering, etc. The basic features of the process are advective and dispersive particle transport and particle capture by the porous medium.

Particle transport in porous media is determined by advective flow of carrier water and by hydrodynamic dispersion in micro-heterogeneous media. Thus, the particle flux is the sum of advective and dispersive fluxes. Transport of particles in porous media is described by an advection–diffusion equation and by a kinetic equation of particle capture. Conventional models for deep bed filtration take into account hydrodynamic particle dispersion in the mass balance equation but do not consider the effect of dispersive flux on retention kinetics.

In the present study, a model for deep bed filtration with particle size exclusion taking into account particle hydrodynamic dispersion in both mass balance and retention kinetics equations is proposed. Analytical solutions are obtained for flows in infinite and semi-infinite reservoirs and in finite porous columns. The physical interpretation of the steady-state flow regimes described by the proposed and the traditional models favours the former.

Comparative matching of experimental data on particle transport in porous columns by the two models is performed for two sets of laboratory data.

Keywords: Deep bed filtration; Dispersion; Suspension; Governing equations; Modelling; Porous media; Emulsion

1. Introduction

Severe injectivity decline during sea/produced water injection is a serious problem in offshore waterflood projects. The permeability impairment occurs due to capture of particles from injected water by the rock. The reliable modelling-based prediction of injectivity decline is important for the injected-water-treatment design, for injected water management (injection of sea- or produced water, their combinations, water filtering), etc.

The formation damage induced by penetration of drilling fluid into a reservoir also occurs due to particle capture by rocks and consequent permeability reduction. Other petroleum applications include sand production control, fines migration and deep bed filtration in gravel packs.

* Corresponding author. Tel.: +55 22 27733391; fax: +55 22 27969734.
E-mail addresses: pavel@lenep.uenf.br, pavel.russia@gmail.com (P. Bedrikovetsky).

0920-4105/$ - see front matter
doi:10.1016/j.petrol.2005.11.010

The basic equations for deep bed filtration taking into account advective particle transport and the kinetics of particle retention, and neglecting hydrodynamic dispersion have been derived essentially following the filtration equation proposed by Iwasaki (1937). A number of predictive models have been presented in the literature (Sharma and Yortsos, 1987a,b,c; Elimelech et al., 1995; Tiab and Donaldson, 1996, Khilar and Fogler, 1998; Logan, 2001). The equations allow for various analytical solutions, which have been used for the treatment of laboratory data and for prediction of porous media contamination and clogging (Herzig et al., 1970; Pang and Sharma, 1994; Wennberg and Sharma, 1997; Harter et al., 2000; Bedrikovetsky et al., 2001, 2002).

However, particle dispersion in heterogeneous porous media is important for both small and large scales (Lake, 1989; Jensen et al., 1997). The typical core sizes in laboratory experiments are small, and hence the Peclet number is relatively high. The typical dispersivity values for large formation scales are high, and consequently the Peclet number may also take high values. The Peclet number for either situation may amount up to 10–20.

The effect of dispersion on deep bed filtration is particularly important near to wells, where the dispersivity may already arise to the bed scale, and the formation damage occurs in one two-meter neighbourhood. Therefore, several deep bed filtration studies take into account dispersion of particles (Grolimund et al., 1998; Kretzschmar et al., 1997; Bolster et al., 1998; Unice and Logan, 2000; Logan, 2001; Tufenkji et al., 2003). A detailed description of such early work is presented in the review paper by Herzig et al. (1970). The models developed account for particle dispersion in the mass balance for particles but do not consider the dispersion flux contribution to the retention kinetics.

In the present study, the proposed deep bed filtration model takes into account dispersion in both the equation of mass balance and in that of capture kinetics. Several analytical models for constant filtration coefficient and for dynamic blocking filtration coefficient have been developed. If compared with the traditional model, the proposed model exhibits more realistic physics behaviour. The difference between the traditional and proposed model is significant for small Peclet numbers.

The structure of the paper is as follows. In Section 2 we formulate the corrected model for deep bed filtration of particulate suspensions in porous media accounting for hydrodynamic dispersion of suspended particles. The dispersion-free deep bed filtration model is presented in Section 3 as a particulate case of the general system with dispersion. The analytical models for flow in infinite and semi-infinite reservoirs for constant filtration coefficient are presented in Sections 4 and 5, respectively. An analytical solution for deep bed filtration in semi-infinite reservoirs with the fixed inlet concentration is given in Section 6. Analytical steady state solution for laboratory coreflood in discussed in Section 7. The analytical models allow for laboratory data treatment (Section 8). Travelling wave flow regimes for dynamic blocking filtration coefficient are described in Section 9. In Section 10, three dimensional equations for deep bed filtration with dispersion are derived. Mathematical details of the derivations are presented in Appendices. Dimensionless form of governing equations and initial-boundary conditions are given in Appendix A. The transient solutions for flow in infinite and semi-infinite reservoirs and constant filtration coefficient are derived in Appendices B, C and D. Appendix E contains derivations for steady state solution in a finite core. Appendix F contains derivations for travelling wave flow.

2. Model formulation

Let us derive governing equations for deep bed filtration taking into account particle dispersion. The usual assumptions of constant suspension density and porosity for low particle concentrations are adopted. The balance equation for suspended and retained particles (Iwasaki, 1937; Herzig et al., 1970) is:

$$\frac{\partial}{\partial t}(\phi c + \sigma) + \frac{\partial q}{\partial x} = 0 \tag{1}$$

Here, the concentration c is a number of suspended particles per unit volume of the fluid, and the retained particle concentration σ is a number of captured particles per unit volume of the rock.

The particle flux q consists of the advective and dispersive components:

$$q = Uc - D\frac{\partial c}{\partial x} \tag{2}$$

$$D = \alpha_D U \tag{3}$$

Here the dispersion coefficient D is assumed to be proportional to the flow velocity U, and the proportionality coefficient α_D is called the longitudinal dispersivity (Lake, 1989; Nikolaevskij, 1990; Sorbie, 1991).

Let us consider the following physical model for the size exclusion particle capture in porous media (Santos and Bedrikovetsky, 2005). Particles are not captured during flow through the pore system, but there is a