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Time-dependent matrix-fracture shape factors for partially and completely immersed fractures

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Abstract

The modeling of multiphase flow in fractured porous media relies on the accurate description of fracture-to-matrix transfer of water. Calculation of this rate, within dual-continuum models, depends on matrix-fracture transfer functions incorporating the so-called shape factor. Typically, matrix-to-fracture transfer functions are obtained by assuming all fractures to be instantaneously immersed in water (instantly-filled), with a uniform fracture pressure distribution under pseudo-steady state conditions. The result is constant, time-independent, shape factors. Clearly, this is not necessarily true. Partially immersed fractures and other unsteady-state conditions do not lead to constant shape factors. A new time-dependent matrix-fracture transfer shape factor formulation and transfer functions for both filling- and instantly-filled fracture transfer are derived based on dimensional analysis of experimental data. The dimensional analysis of full-physics data avoids simplifications that may lead to expressions that do not represent accurately matrix-fracture transfer. The new shape factor carries information about the transient behavior of the water saturation, S_w, and so it leads to more accurate description of the matrix-fracture transfer. Good agreement was found between experimental data, an analytical model, and a proposed modified dual-porosity formulation with the new time-dependent shape factor and transfer function.

Keywords: Imbibition; Capillarity; Fractured Porous Media; Dual Continua

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1. Introduction

Fractured systems are usually modeled by means of a dual-porosity or dual-permeability formulation. One of the chief features of such formulations is the evaluation and modeling of the matrix-fracture mass transfer. Rock matrix blocks contribute the main portion of the reservoir pore volume, but they have much smaller permeabilities than do the fractures. Flow between the matrix block and the fracture is fundamental to the productivity of fractured formations, and such flow is estimated numerically with a matrix-fracture transfer function incorporating a shape factor.

Unfortunately, the physics of matrix-to-fracture transfer have not been elucidated clearly. Because previous studies have emphasized various physical aspects, the transfer functions available in the literature predict a variety of recovery behaviors. This paper is motivated by a need for expressions that account for fractures that are partially or totally immersed with water under unsteady conditions. The formulation incorporates parameters that are obtained either in the laboratory or the field with some degree of certainty. Specifically, the contribution of

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the paper is the development of a more general time-dependent methodology for matrix-fracture transfer that accounts for volumetric expansion of fluids as well as capillary imbibition. Results are cast in the form of shape factors so as to be consistent with current-day simulation practice.

This paper proceeds with an overview of shape factors and the asymptotic behavior of prior formulations. It continues by summarizing recent experimental results and an analytical model for capillary imbibition (Rangel-Germán and Kovscek, 2002; Rangel-Germán, 2002). The introductory and review material is relatively detailed to allow a smooth description and definition of a new methodology to obtain shape factors and transfer functions. Finally, examples compare the results among experimental data, analytic model, and the modified dual-porosity formulation.

2. Current matrix-fracture transfer modeling

The dual porosity idealization for describing flow in fractured media was introduced by Barenblatt et al. (1960) and later by Warren and Root (1963). The approach divides the domain into separate, interacting continua that model matrices and fractures. Representation of matrix-to-fracture transfer, and vice versa, is fundamental to the approach. For single-phase, pseudo-steady state flow the transfer rate reads

$$\tau = \frac{\sigma k V}{\mu}\left(\overline{p_m} - p_f\right) \tag{1}$$

where τ is the flow rate in an element V of bulk reservoir volume, p_m is the volumetric average matrix pressure and p_f is the pressure in the fracture. The shape factor, σ, reflects the geometry of the matrix elements and is time independent for pseudo-steady state conditions. Table 1 compares the variety of shape factors available in the literature.

Significantly, Kazemi et al. (1976) extended the formulation to two-phase flow. The chief assumptions to obtain a shape factor are that the pressure distribution in the fractures surrounding a matrix block is uniform and that a linear pressure gradient exists between the matrix and the fracture. For a cubic matrix rock, σ has the value of $12/L^2$; and for two and three sets of normal fractures, the values of σ are $4/L^2$ and $8/L^2$, respectively. Subsequently, Gilman and Kazemi (1983) extended the shape factor formulation of Kazemi et al. (1976) to represent permeability anisotropy

$$k_m \sigma = 4\left(\frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2}\right) \tag{2}$$

In related studies, Thomas et al. (1983) matched fracture modeling results with oil recovery from water imbibition in single-cell experiments. They found shape factors of 25 and 0.25 for 1- and 10-ft cubic blocks, respectively. This is equivalent to an equation of $25/L^2$ for the shape factor. Coats (1989) and Ueda et al. (1989) closely matched modeling results with laboratory experimental results and fine-grid simulation. They found that Kazemi's shape factors had to be multiplied by factors of at least 2 to 3 for one and two sets of fractures, respectively. That is, actual recovery is significantly more efficient than the predicted by the pseudo-steady state approach.

Table 1 summarizes some of the values reported for σ. There are great discrepancies. This does not mean, however, that there is one single correct value and the rest are incorrect. The discrepancies indicate a need for better understanding of matrix-fracture interaction is required to explain why the values are so different.

Alternative derivations have avoided, at least partially, the pseudo-state assumption by combining the geometrical aspects of the systems with analytical solutions of the pressure diffusion equation. For example,

Table 1
Comparison of shape factors reported in the literature

σL^2								
N	Warren and Root (1963)	Kazemi et al. (1976)	Thomas et al. (1983)	Ueda et al. (1989)	Coats (1989)	de Swaan (1990)	Chang (1993)	Lim and Aziz (1995)
1	12	4		8	8	12	π^2	π^2
2	32	8		24	16		$2\pi^2$	$2\pi^{2\mathrm{a}}$, 18.17[b]
3	60	12	25		24	60	$3\pi^2$	$3\pi^{2\mathrm{a}}$, 25.67[c]

[a] from Newman product.

[b] cylindrical approximation.

[c] spherical approximation.