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Unit commitment for a compressor station by mixed integer linear programming

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ABSTRACT

When operating a compressor station, given its mass flow rate, inlet pressure and temperature, and discharge pressure, dispatchers need to decide which compressors to run and at what flow rates, i.e., the operating scheme of the station, to cut its energy costs. This paper addresses this problem under unsteady states. This means that at least one of the four given operating parameters is time-dependent, and therefore so is the operating scheme. The key constraints of the problem are the minimum uptime and downtime of each compressor, which interconnect the operating schemes at each time step and complicate the problem. The energy cost of a compressor unit is almost a linear function of its flow rate for a given inlet pressure, inlet temperature, and discharge pressure. Therefore, the optimization problem was formulated as a mixed integer linear programming (MILP) model, which was solved by CPLEX. The optimal operating schemes given by CPLEX were simulated to reevaluate the objective function, and the error of the linearized energy cost model was shown to be within 5%. The recalculated objective function values were 0.22%–1.18% higher than those of the true optimum. However, the MILP method was 0.49–64.95 times faster than the dynamic programming approach yielding global optimal solutions.

1. Introduction

Pipelines are widely used to transport natural gas over long distances around the world. As gas flows through a pipe segment, its pressure drops. Compressor stations located along a gas pipeline compensate for this pressure drop by consuming large amounts of energy to compress the gas. Their energy costs make up the majority of gas pipeline operating costs. Hence, power optimization of compressor stations has attracted great interest.

In most cases, a compressor station consists of several compressors in parallel. Knowing the mass flow rate, the inlet pressure and temperature, and the discharge pressure of a compressor station, dispatchers need to decide which units to run and at what flow rates, i.e., the station operating scheme, to minimize station energy cost. Some constraints on compressor operation in parallel must also be satisfied.

The steady-state version of this problem has attracted wide interest (Botros et al., 2011; Carter, 1996; Jenicek and Kralik, 1995; Wright et al., 1998). Under steady state, the parameters and the operating scheme of a compressor station are not affected by time. Several efficient steady-state solution methods are available, including gradient-based algorithms (Paparella et al., 2013; Xenos et al., 2014, 2015), mixed integer linear programming (Carter, 1996), simulated annealing algorithms (Wright et al., 1998), genetic algorithms (Hawryluk et al., 2010; Mahmoudimehr and Sanaye, 2014), and dynamic programming (Zhang et al., 2014).

However the unsteady-state version of this problem has attracted little interest. This kind of problem originates from optimizing gas pipeline operation under unsteady states. The problem has two levels: a pipeline level and a station level. The pipeline level adjusts the discharge pressure of each compressor station and simulates gas flow in pipe segments. Given the mass flow rate, the inlet pressure and temperature, and the discharge pressure of a compressor station, the station level determines the operating scheme that will minimize overall station energy consumption. Note that the four given operating parameters are likely to be time-dependent, and hence the operating scheme will be time-dependent also. This means that in addition to the constraints mentioned earlier for the steady-state problem, the minimum uptime and downtime of each compressor unit must also be

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considered. Once a compressor has been started up, its minimum uptime requires that it must run at least for a prescribed period of time before it is shut down. The minimum downtime of a compressor has a similar meaning. These extra constraints interconnect the station operating schemes at each time step, complicating the problem.

This problem is similar to the unit commitment (UC) problem in the power generation industry. For clarity, the latter will be referred to here as the generator UC problem and the former as the compressor UC problem. The generator UC problem has attracted great interest (Padhy, 2004; Sen and Kothari, 1998; Yamin, 2004), and the methods used to solve it include priority lists, dynamic programming, mixed integer linear programming (Carrión and Arroyo, 2006; Garver, 1962; Jabr, 2012), and genetic algorithms (Dang and Li, 2007; Kazarlis et al., 1996; Pavez-Lazo and Soto-Cartes, 2011; Sun et al., 2006; Yang et al., 1996). The generator UC problem is usually solved as a separate problem. However, the compressor UC problem is a sub-problem that must be solved hundreds to thousands of times to optimize gas pipeline operation. Hence, the method used to solve the compressor UC problem should be efficient.

The following sections describe the formulation of a mixed integer nonlinear programming (MINLP) model to describe the compressor UC problem. This model was then approximated by a mixed integer linear programming (MILP) model, which was solved by CPLEX. This approach was tested on three compressor stations. Analysis of the results included approximation error analysis of the MILP model and a comparison between the approximate optimal solutions and the true optimal ones.

2. Mathematical model

The aim of a compressor UC problem is to minimize the accumulated energy cost of a compressor station during a period of time, illustrated as Eq. (1), where T is the period of time, N the number of compressors in the station, Q the mass flow rate passing through a compressor in the station, P_{in} and T_{in} the inlet pressure and temperature of the station respectively, P_d the discharge pressure of the station, and f the energy cost of a compressor unit. Note that for a specific compressor UC problem, P_{in}, T_{in}, and P_d are given. Hence, Eq. (1) can be simplified to Eq. (2), and the latter is computed according to the trapezoid rule, illustrated as Eq. (3), where $\Delta\tau$ is the time step and T_{Num} the number of time steps. Note also that the energy cost of a compressor unit is usually computed by simulation (Zhang et al., 2014).

$$\min \sum_{n=1}^{N} \int_{\tau=0}^{T} f_n\left(Q_n(\tau), P_{in,n}(\tau), T_{in,n}(\tau), P_{d,n}(\tau)\right) d\tau \tag{1}$$

$$\min \sum_{n=1}^{N} \int_{\tau=0}^{T} f_n(Q_n(\tau)) d\tau \tag{2}$$

$$\min \sum_{n=1}^{N} \sum_{t=1}^{T_{Num}} \frac{\Delta\tau}{2} \left[f_n(Q_{n,t-1}) + f_n(Q_{n,t})\right] \tag{3}$$

The problem is subject to two classes of constraints corresponding to the station as a whole and to each unit in the station. The first class expresses the flow rate balance of the station, illustrated as Eq. (4), where Q_{cs} is the total mass flow rate of the station. The second class includes Eqs. (5)–(8). Eq. (5) defines the feasible flow rate region of a compressor, where Q_{min} and Q_{max} are its smallest and largest flow rate under specific conditions P_{in}, T_{in}, and P_d, and u represents the compressor state. If the compressor is on, $u = 1$; otherwise $u = 0$. Eqs. (6) and (7) describe the minimum uptime and minimum downtime constraints of a compressor (Moritz, 2007). In the two equations, L is the minimum uptime of the compressor and l its minimum downtime. By introducing an extra kind of variable v_t, Rajan and Takriti (2005) have shown that Eqs. (9)–(12) describes the convex hull defined by Eqs. (6)–(8). In these equations, v_t represents whether a compressor is started up at time step t. It is equal to 1 if and only if the compressor is offline at time step t-1 and is online at time step t. Finally, Eqs. (13) and (14) are the initial conditions. It is assumed that the compressors have been online or offline for enough time. Hence, in the first time step, each of them can be started up or shut down.

$$\sum_{n=1}^{N} Q_{n,t} = Q_{cs,t}, \quad t = 1, \ldots, T_{Num} \tag{4}$$

$$u_{n,t} Q_{min,n,t} \le Q_{n,t} \le u_{n,t} Q_{max,n,t}, \quad n = 1, \ldots, N, t = 1, \ldots, T_{Num} \tag{5}$$

$$u_{n,t} - u_{n,t-1} \le u_{n,j}, \quad t+1 \le j \le \min\{t+L-1, T\} \quad n = 1, \ldots, N, t = 1, \ldots, T_{Num} - 1 \tag{6}$$

$$u_{n,t-1} - u_{n,t} \le 1 - u_{n,j}, \quad t+1 \le j \le \min\{t+l-1, T\} \quad n = 1, \ldots, N, \ t = 1, \ldots, T_{Num} - 1 \tag{7}$$

$$u_{n,t} = 0, 1, \quad n = 1, \ldots, N, t = 1, \ldots, T_{Num} \tag{8}$$

$$\sum_{i=t-L+1}^{t} v_{n,i} \le u_{n,t}, \quad n = 1, \ldots, N, t = L, \ldots, T_{Num} \tag{9}$$

$$\sum_{i=t-l+1}^{t} v_{n,i} \le 1 - u_{n,t-l}, \quad n = 1, \ldots, N, t = l, \ldots, T_{Num} \tag{10}$$

$$v_{n,t} \ge u_{n,t} - u_{n,t-1}, \quad n = 1, \ldots, N, t = 1, \ldots, T_{Num} \tag{11}$$

$$v_{n,t} \ge 0, \quad n = 1, \ldots, N, t = 1, \ldots, T_{Num} \tag{12}$$

$$u_{n,0} = u'_{n,0}, \quad n = 1, \ldots, N \tag{13}$$

$$q_{n,0} = q'_{n,0}, \quad n = 1, \ldots, N \tag{14}$$

Eqs. (4) and (5) and Eqs. (9)–(14) describe the compressor UC problem. Compared with the generator UC problem, it is less constrained and involves fewer units. However, its objective function is usually computed by simulation, as stated above, whereas that of the generator UC problem is quadratic. This makes the current problem as hard as the generator UC problem.

3. MILP approach

Note that only the objective function in the previous model is nonlinear, and many powerful MILP solvers are available. Therefore, the objective function was approximated by linear functions, and the previous MINLP model was reformulated as an MILP model. As illustrated in Eq. (3), the objective function is related to the energy cost of a compressor unit at a given time step. Given the inlet pressure, the inlet temperature, and the discharge pressure of a