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A similarity method approach for early-transient multiphase flow analysis of liquid-rich unconventional gas reservoirs

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ABSTRACT

Despite their increased role in the global energy supply, liquid-rich unconventional gas (LRG) resources present a number of technical challenges. Among them, traditional production data analysis methods fail to successfully estimate and forecast production behavior in these systems. This failure is directly related to the extensive early-transient infinite-acting behavior exhibited by these systems, and the added complexities involved with liquid dropout and ensuing multi-phase flow of gas and condensate. Traditional approaches are strongly biased toward single-phase and boundary-dominated analysis; and when multiphase flow is considered, required input data often include laboratory-estimated pressure-saturation data and/or producing gas-oil-ratio data. In the present work, a novel extension of the similarity variable transformation method is developed to forecast production behavior in these LRG systems. The methodology uses the black-oil fluid formulation and considers linear and radial flow regimes under constant bottom hole pressure (BHP) and constant gas flow rate well conditions. Using this method, the system of governing partial differential equations is reduced to a system of ordinary differential equations solved by well-known Runge–Kutta techniques without the need for linearization. It is demonstrated that reservoir pressure and saturation behavior can be forecast simultaneously, thereby eliminating the need for pressure-saturation relationship or producing gas-oil-ratio data as inputs to the model. In all cases explored, the similarity results compared well to numerically generated reservoir data for a variety of well BHP and gas flow rate specifications. Calculated well production metrics also successfully matched data sets, indicating that this approach can be straightforwardly extended to estimate production metrics of interest during early transient conditions, such as liquid and gas production rates or gas-oil-ratio. Results strongly suggest that the method developed here provides a rapid and robust alternative to numerical simulation for forecasting of LRG reservoir systems.

1. Introduction

Exploitation of unconventional liquid-rich gas (LRG) reservoirs, including shale gas, tight gas, and coalbed methane, is becoming increasingly important to the energy supply in North America. From 2005 to 2013, in the United States alone, natural gas production increased by 35%, accounting for an increase from 23% to 28% of the natural gas share of total energy consumption in the USA during that time (EIA, 2015). Despite the increase in demand, unconventional LRG reservoirs exhibit a number of technical challenges that are fundamentally different from conventional reservoirs. These challenges are directly related to the extensive early-transient infinite-acting behavior exhibited by LRG systems, the presence of liquid dropout, and ensuing multi-phase flow of gas and condensate. Additional complexities in unconventional systems include apparent permeability (Javadpour, 2009), flow regimes resulting from well completion (*e.g.* multi-stage fractured horizontal wells) (Clarkson, 2013a), and behavior of hydrocarbons in place as a result of nano-scale pores (Jin and Firoozabadi, 2015; Loucks et al., 2009). Because of these fundamental differences, production analysis methods that are traditionally employed for conventional reservoir analysis fail to successfully estimate and forecast production behavior in LRG systems, and new formulations are needed for unconventional reservoir system analysis (Clarkson, 2013a, 2013b; Clarkson et al., 2012).

Production decline analysis (PDA) methods aim to quantitatively

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interpret well production data and generate predictions of fundamental characteristics of the reservoir in question, often by invoking solutions to the governing reservoir flow equations (Ilk et al., 2010). Information that can be estimated using PDA techniques includes estimated ultimate recovery (EUR), original gas in place (OGIP), fracture characteristics, reservoir permeability, and/or stimulated reservoir volume. For gas reservoir analysis, PDA techniques traditionally assume single-phase gas flow and many do not consider liquid-phase dropout that is often observed during the production life of LRG reservoirs. In many studies, analysis of single-phase gas flow routinely involves linearization of the non-linear gas flow equations by expressing the equations in terms of pseudopressure (Al-Hussainy et al., 1966) and pseudotime (Agarwal, 1979). This approach transforms the governing equations to a form that can be solved with existing methods for slightly compressible fluid flow, and has been traditionally employed to develop type-curve or straight-line relationships for boundary dominated flow (BDF) in both linear and radial regimes (Fetkovich, 1980; Fraim, 1987; Wattenbarger et al., 1998). However, these approaches are strongly biased toward single-phase and boundary-dominated analysis. For early-transient analysis of single-phase gas flow, an extension of this approach to infinite-acting analysis has been proposed in which pseudovariables are evaluated in a “region of influence” (Anderson and Mattar, 2005; Nobakht et al., 2011). To account for multiphase flow effects, two-phase pseudopressure and pseudotime have been introduced (Camacho and Raghavan, 1989; Sureshjani and Gerami, 2011) and also applied to early-transient linear flow coupled with the region of influence concept (Behmanesh et al., 2015b). Despite these efforts, the two-phase pseudovariable transformation approach for multi-phase flow presents an important limitation: pressure-saturation relationship (typically estimated in the laboratory and/or from producing gas-oil-ratio data) is required to be obtained *a priori* in order to evaluate pseudopressure and pseudotime. An additional difficulty is that laboratory-estimated pressure-saturation results could misrepresent the actual behavior of the reservoir, especially in the case of LRG reservoirs (Whitson and Sunjerga, 2012).

An alternate approach to early-transient “region of influence” analysis is the development of analytical and semi-analytical solutions to the reservoir flow equations by employing the similarity method (also referred to as Boltzmann transformation) (Ayala and Kouassi, 2007; Boe et al., 1989; Kouassi and Ayala H. 2009; Zhang et al., 2014). The similarity method transforms the governing partial differential equations (PDEs) for fluid flow into ordinary differential equations (ODEs) that are written in terms of a single independent variable combining time and space (Doughty and Pruess, 1990; O'Sullivan, 1981). This method provides two distinct advantages in that well-known ODE solvers can be employed to solve the resulting system of equations and linearization of the equations is not required. This approach has been specifically applied to unconventional reservoir analysis by a number of studies where the resulting semi-analytical solutions were demonstrated to match synthetically generated cases in a number of flow regimes and well conditions (Behmanesh et al., 2015b; Zhang et al., 2014). Similarity transformations have also been undertaken to solve multi-phase flow equations in gas condensate reservoirs; however, in those studies, pressure was the only dependent variable considered and pressure-saturation data was also required (Behmanesh et al., 2015b; Boe et al., 1989), resulting in the same limitations described above. Likewise, Tabatabaie (2014) explored a similarity transformation restricted to linear constant BHP and classical black-oil multiphase systems (*i.e.* $R_v = 0$). To our knowledge, no analytical production analysis method developed for liquid-rich gas systems to date has considered pressure and saturation behavior as separate dependent variables that can be resolved simultaneously and without *a priori* availability of gas-oil ratio data.

The present work aims to fill this knowledge gap by extending a modification of the similarity method, previously developed in the context of single-phase unconventional gas reservoirs (Zhang et al., 2014), to multi-phase LRG reservoirs. Here, similarity solutions to the governing equations for multi-phase flow were derived for two inner boundary conditions (constant bottom hole pressure (BHP) and constant gas flow rate) in each of the two traditional flow regimes (linear and radial). In all cases, pressure and saturation were simultaneously considered as separate dependent variables, thereby eliminating the potential issues associated with inputting pressure-saturation data. Following case-by-case analytical development, similarity method results were compared to synthetic data generated by a finely-gridded numerical simulation. In each flow regime and boundary condition, the model's ability to predict PDA metrics of interest (*e.g.* gas flow rate and well bottom hole pressure) was also explored to demonstrate the capabilities of the proposed method.

2. Method of approach

In this study, semi-analytical similarity solutions to the modified black oil model for liquid-rich gas reservoirs were developed for four scenarios: constant BHP and constant gas flow rate in each of the linear and radial regimes. In all cases, separate analytical developments were undertaken, as the similarity solutions are dependent on initial and boundary conditions. Following solution derivations for each case, presented in the following sections, simulations were conducted under two specified well conditions (constant BHP or constant rate) for each of two flow regimes (linear and radial). Linear and radial flow regimes were considered here since they represent the two end-members of realistic flow regimes in unconventional gas production systems. All simulations were conducted using a single set of PVT properties which were generated by applying a Peng–Robinson equation of state to a ten-component gas condensate fluid through a standard constant-volume-depletion (CVD) calculation (Fig. 1A) (Peng and Robinson, 1976). A single set of relative permeability curves was implemented for all case studies undertaken (Fig. 1B). Relevant reservoir parameters used in this investigation are presented in Table 1, which consider a liquid-rich gas fluid found initially saturated at dewpoint pressure. To generate test case data and validate similarity solutions, corresponding numerical simulations were conducted using a commercial black-oil simulator (CMG-IMEX, Version 2014.10, Computer Modelling Group, Ltd., Calgary, Alberta, Canada).

For multiphase flow analysis using a black oil fluid formulation, the governing partial differential equations for surface gas and stock tank oil are written as, respectively:

$$\nabla \cdot \left[\left(\frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) \nabla p \right] = \frac{\phi}{k} \frac{\partial}{\partial t} \left(\frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \tag{1}$$

$$\nabla \cdot \left[\left(\frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) \nabla p \right] = \frac{\phi}{k} \frac{\partial}{\partial t} \left(\frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \tag{2}$$

For this preliminary study, capillary pressure and rock compressibility have been neglected to showcase the fundamentals of the proposed methodology, a routine assumption in multiphase production data analysis methods for unconventional reservoirs (Behmanesh et al., 2015b; Hamdi et al., 2015). These additional non-linearities, however, may be also incorporated without changes to the underlying similarity transformation technique. The pressure, p, and oil saturation, S_o, functions are treated and solved