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A theoretical study of the critical external pressure for casing collapse

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ABSTRACT

A new theoretical casing collapse model is proposed on the basis of a refined reduced modulus calculation method. The reduced modulus of considers the co-existence of the elastic and plastic regions on the casing cross section. The collapse pressure for an ideal circular casing is expressed as a weighted harmonic sum of the critical elastic pressure and casing yielding pressure. Three kinds of casing imperfections including the wall thickness variation, ovality and non-uniform external pressure on casing collapse are further considered to amend the ideal casing collapse model. The equivalent wall thickness due to wall thickness variation is introduced, and the critical elastic pressure and yielding pressure under the effects of initial ovality and non-uniform external pressure are obtained. On this basis, the collapse pressure for an oval casing with thickness variation under non-uniform external pressure is obtained with superposition principle. The effects of axial force, bending moment and torque on collapse pressure are included with the reduced yielding point. All the above factors are integrated into a unified casing collapse model. At last, the comparison between the new model and API specification is further discussed. The results indicate that the new model gives a more sophisticated description of casing collapse problem. Introducing the casing imperfections improves the consistency between the theoretical and experimental results at the transition between elastic and plastic ranges. The non-uniform external pressure plays the most important role in the casing collapse problem. The existence of non-uniform external pressure, axial force, bending moment and torque also reduces the critical casing collapse pressure a lot.

1. Introduction

Casing collapse is an important issue during the oil well completion process. The pioneering casing collapse models were focused on elastic stability analyses of the circular tubing with external pressure (Timoshenko and Gere, 1963). The elastic collapse model is applicable to the thin-walled tubes (large diameter-wall thickness ratio (D/t)) but not to the thick-walled tubes (large diameter-wall thickness ratio (D/t)). One reason for the inconsistency is that part of the casing has already yielded before the external pressure reaches the critical elastic buckling value. To extend the application scope, the hoop stress formula is introduced as the critical plastic pressure for thick-walled tubes. The combination of the elastic stability formula and hoop stress roughly depicts the critical collapse external loads for casing collapse. However, the calculated results from the extended model are still not in good consistency with the measured data from experiments, especially on the transition section between elastic and plastic collapses. To overcome this shortcoming, Clinedinst (Clinedinst, 1939) and Holmquist (Holmquist and Nadai, 1939) introduced the definition of "reduced modulus" and proposed a new formula of the critical plastic pressure. Their results show that the critical collapse pressure is closely related to the shape of stress–strain curve. Zhao (Zhao, 1978) recommended a more sophisticated expression to calculate the "reduced modulus" and deduced an analytical expression of the critical collapse pressure for the casing with initial ovality. In recent years, Issa (Issa and Crawford, 1993) and Huang (Huang & et al., 2000) studied the effects of geometric imperfections (initial ovality shape and wall thickness variation) on the collapse pressure and inversed some useful empirical expressions from numerical results. Li (Li & et al., 2012), Yin (Yin and Gao, 2014) and El-Sayed (El-Sayed and Khalaf, 1992) studied the casing collapse under non-uniform external pressure. Recent studies (Main, 1939; Kuriyama & et al., 1992) indicate that other external forces such as bending moment, axial force and torque also have a

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significant effect on the casing collapse pressure. Therefore, an integral model should be proposed by taking these factors into a comprehensive model.

In this paper, the relevant factors are divided into four categories: (1) the stress–strain curve, (2) casing imperfections, (3) non-uniform external pressure and (4) other external forces including axial force, bending moment and torque. The critical casing collapse pressure is refined based on the ideal elastic–plastic stress strain curve. With the new model, the effect of casing imperfections on the critical pressure is discussed and the influences of non-uniform pressure on circular and oval casings are compared. Besides, the modification of the critical pressure under other external forces is given.

2. Elastic and plastic collapse models

2.1. Previous results

It is well known that a straight column with rectangular sections of unit width keeps its initial configuration if the axial compression is smaller than a certain value, and buckles after the axial compression exceeds this value. The critical axial compression for column buckling is expressed in the following form:

$$\sigma = \frac{E^{*}\pi^{2}}{12}\left(\frac{h}{l}\right)^{2} \tag{1.1}$$

Where σ is the average axial compression stress, l is length of the column, h is the height of the cross section. If the axial stress σ is smaller than the proportional limit σ_p, E^* designates Young's modulus of elasticity E. However, if the axial stress σ is larger than the proportional limit, E^* designates the reduced modulus calculated by

$$E^{*} = \frac{4EE'}{\left(\sqrt{E}+\sqrt{E'}\right)^{2}} \tag{1.2}$$

Where E' represents the tangent modulus and is equal to the slope $d\sigma/d\varepsilon$ of the stress strain curve.

With external pressure, a tube may be in the elastic state or plastic state. Elastic state means the deformation of the tube is within the elastic range while plastic state indicates that part or whole of the tube is in the plastic range as is shown in Fig. 1.

Similar to the column buckling problem, the critical average tangential stress for tube collapse is expressed in the following form:

$$\sigma_t = \frac{E^{*}}{(1-\nu^{2})}\frac{D/t}{(D/t-1)^{3}} \tag{1.3}$$

and the critical external compression for tube collapse is expressed by

$$p_c = \frac{2E^{*}}{(1-\nu^{2})}\frac{1}{D/t(D/t-1)^{2}} \tag{1.4}$$

Where t is the thickness of the tube, D is the outer diameter of the tube, ν is the Poisson's ratio.

According to Eq. (1.1) and Eq. (1.3), the average stress for the column is analogous to that of the tube. However, Zhao (Zhao, 1978) pointed out that the above analogy between the column and tube is only satisfied for tubes with thin walls and quite thick walls rather than for the transitional case namely tubes with slightly thick walls. Zhao presented a more accurate model to depict the stress distribution on the cross section of tubes and introduced a novel reduced modulus. However, Zhao's theoretical derivation results are too complicated to be applied in engineering practice. Therefore, a simple but effective model for tube collapse is proposed.

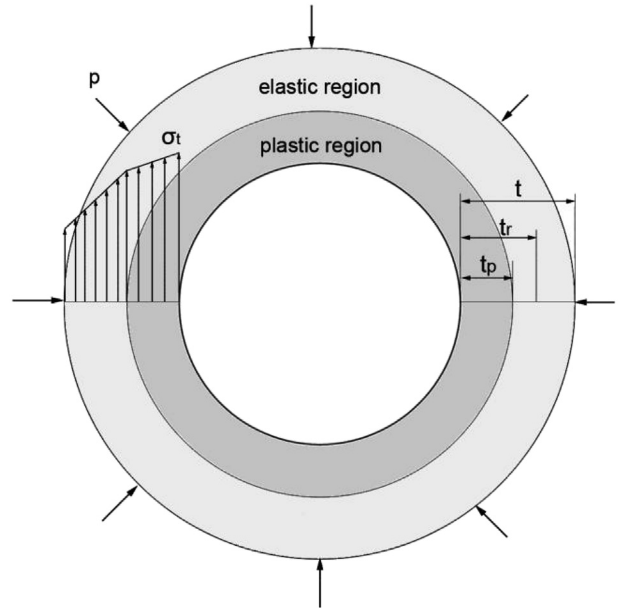

Fig. 1. Elastic region and plastic region in a tube with external pressure.

2.2. New model

For a tube compressed by an external pressure p, the tangential stress σ_t can be obtained with Lame's formula, namely

$$\sigma_t = \frac{p(D/t)^{2}}{4(D/t-1)}\left(1+\frac{(D/2-t)^{2}}{(D/2-t+t_r)^{2}}\right) \tag{1.5}$$

Where t_r is the radial distance between the current point and the tube inner surface.

If the external pressure exceeds a certain value, the inner surface of the tube switches to the plastic state. Assume that the stress strain curve conforms to the ideal elastic–plastic model and the proportional limit σ_p is equal to the yielding strength σ_s as is shown in Fig. 2.

The yielding strength σ_s can be expressed as the tangential

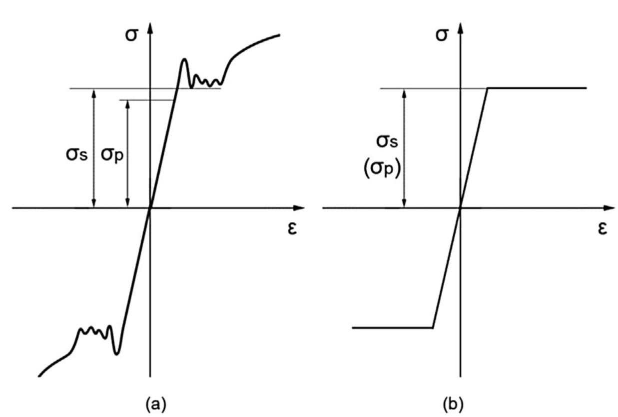

Fig. 2. Stress strain curve: (a) actual stress strain curve and (b) ideal stress strain curve.