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Diagnosis of nonlinear reservoir behaviour for correctly applying the superposition principle and deconvolution

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A R T I C L E I N F O

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A B S T R A C T

The superposition principle is the fundamental theory of pressure-transient analysis. Many pressure analysis methods, such as pressure build-up analysis and deconvolution, are based on the superposition principle. Deconvolution, which can provide the equivalent constant-rate pressure response from variable-rate pressure data, has become more popular in recent years. Mathematically, the superposition principle is only valid in a linear system. However, many reservoir behaviours can make the system nonlinear and the superposition principle invalid, such as the time-dependent skin factor and permeability, multiphase flow and non-Darcy flow. Erroneously applying the superposition principle and related methods will lead to incorrect analysis results and inappropriate production decisions. To reduce the analysis uncertainties and obtain the correct reservoir information, diagnosing the nonlinear reservoir behaviour and selecting the appropriate pressure analysis method are important. In this paper, nonlinear reservoir behaviours are diagnosed from transient pressure data with the wavelet transform. A defined diagnosis function can effectively diagnose the nonlinear reservoir behaviours. As studies have proved, the diagnosis function is constant in the linear system and time varying in the nonlinear system. Based on the diagnosis result, the nonlinear system can be linearized in a short time period using the sliding window technique, and the superposition principle and related methods can be correctly applied.

1. Introduction

For the short-time traditional well test, flow conditions and reservoir properties are nearly constant. In the single oil phase flow reservoir, the linear diffusivity equation governing fluid flow and constant boundary conditions ensure the superposition principle is valid. Conventional pressure analysis methods are applicable, such as pressure build-up analysis and deconvolution. Deconvolution can provide an equivalent constant-rate pressure response from measured variable-rate pressure data and has received considerable attention in the last decade (Gringarten, 2010). The time interval of deconvolution is longer than that of a traditional well test, and the radius of investigation is extended. Deconvolution is an ill-posed inverse problem, which means that small errors of input can lead to large changes in output. Deconvolution was not applied widely in the oil industry until the publication of the stable algorithm using the nonlinear total least squares method (von Schroeter et al., 2001, 2004). Based on this method, Levitan (2005) and Levitan et al. (2006) made some improvements with practical considerations. Ilk et al. (2006) presented B-spline deconvolution and applied this algorithm on the gas well test. With the installation of permanent down-hole pressure and flow-rate measure systems, deconvolution becomes more important because it can process pressure and rate data simultaneously and can obtain the underlying reservoir properties, which change with time (Onur et al., 2008).

Mathematically, the superposition principle and deconvolution can only be used in a linear system. Considering the long-term and complex nature of reservoir production, reservoir properties and flow conditions are always changing with time, such as permeability change and multiphase flow after water/gas breakthrough, which will make the diffusivity equation highly nonlinear and the superposition principle invalid. As a result, transient pressure is complex to analyse with conventional methods, and the derived reservoir information will have uncertainties when inappropriate pressure-transient analysis methods are used. Kuchuk et al. (2005) noted that any changes in the reservoir model could make deconvolution nonlinear. Levitan (2005) and Houze et al. (2010) demonstrated that deconvolution fails for the changes of the

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wellbore storage and skin case. Ilk et al. (2010) discussed several scenarios where deconvolution is invalid, such as gas flow, multiphase flow, and non-Darcy flow. These nonlinearities usually are caused by the changes in reservoir properties and well conditions. Therefore, diagnosing and evaluating nonlinearity in the reservoir is important. After diagnosis, the superposition principle and related methods can be applied with confidence, and correct reservoir information can be derived from pressure data. However, there are no effective methods for diagnosing nonlinearity. At present, the main diagnosis method is pressure and pressure-derivative analysis in a log–log plot. All pressure build-ups are plotted in the same plot and changes in reservoir properties and flow conditions can be detected, but there are a limited number of pressure build-ups and therefore nonlinearity cannot be diagnosed in real-time. Ilk et al. (2010) and Li et al. (2011) used deconvolution as a diagnosis tool, but there are theory uncertainties when applying the deconvolution algorithm in nonlinear systems. Another effective diagnosis method should be developed.

This paper presents a method of diagnosing nonlinearity with the wavelet transform (WT). The first application of WT in permanent down-hole gauge (PDG) pressure data processing was proposed by Kikani and He (1998) and Athichanagorn et al. (1999). They found that WT is an effective method of processing PDG pressure data, including outlier removal, denoising, and transient identification. The wavelet modulus maxima method is used to identify transient events. With pressure data processed with WT, there are amplitudes of WT coefficients at the time of transient events. Wang (2012) and Wang and Zheng (2013) quantitatively analysed the relationship between the amplitude of WT coefficients and the flow rate change, and an algorithm of calculating unknown flow rates was proposed. Wang and Zheng (2014) used WT to analyse PDG pressure data in the frequency domain to diagnose changes in reservoir properties. This paper will expand this method to diagnose nonlinear reservoir behaviours. Moreover, based on the nonlinearity diagnosis result, the sliding window technique is used to divide pressure history into different time windows and in each time window the nonlinearity is so small that deconvolution can be applied to obtain correct reservoir information.

2. Theory description

2.1. Superposition principle and nonlinear reservoir behaviours

For the well test, if the reservoir is treated as a system, then flow rate and down-hole pressure are the input and output signals, respectively. In a linear system, the wellbore pressure $p_{wf}(t)$ and flow rate $q(t)$ can be described as Duhamel's integral (Van Everdingen and Hurst, 1949):

$$p_{wf}(t) = p_0 - \int_0^t q(\tau)g(t-\tau)d\tau \tag{1}$$

where p_0 is the initial reservoir pressure and $g(t)$ is the reservoir system impulse function, which is determined by the reservoir and well properties. Moreover, $g(t) = \frac{dp_u}{dt}$ and p_u is the rate-normalized pressure response. Calculating p_u with measured pressure $p_{wf}(t)$ and flow rate $q(t)$, this is pressure-rate deconvolution.

Mathematically, Eq. (1) is an expression of the superposition principle and is only theoretically valid in a linear system. If the reservoir is a linear system, the diffusivity equation governing fluid flow in porous media should be linear, and the associated boundary condition should be constant with time. The diffusivity equation is derived from the continuity equation, the Darcy flow equation and the equation of rock and fluid properties, and usually the inner boundary condition has the effect of skin factor and wellbore storage (Muskat, 1934; Hurst, 1934). To satisfy the linear system requirement, the flow is a slightly compressible single-phase flow in porous media, the properties of the porous media are constant with time, the well conditions, such as the wellbore storage and skin factor, are constant and there is no turbulent flow. In practice, many reservoir behaviours can cause a nonlinear diffusivity equation and can change the associated boundary conditions, which leads to a nonlinear system and invalidates the superposition principle. These nonlinear behaviours can be summarized as follows:

- Reservoir formation and fluid properties change
- Flow conditions change, such as multiphase flow and non-Darcy flow
- Well conditions change, such as variable wellbore storage

Different nonlinear reservoir behaviours need appropriate solving methods. These solving nonlinearity methods can be summarized as follows:

- Analytical method, such as Buckley–Leverett equation modelling two-phase flow in porous media (Buckley and Leverett, 1942)
- Numerical method, such as numerical well testing
- Linearization method with parameter transform, such as pseudo-pressure for real gas, and Perrine's method for multiphase flow (Perrine, 1956)
- Approximation method, when reservoir properties change slightly in a short period of time, reservoir can be approximated to be linear

Before solving nonlinear problems in the reservoir, diagnosing and evaluating the nonlinearity in the reservoir is a critical step.

The sliding window technique is widely applied in signal processing and data mining, especially for a long and time-dependent dataset (Luo and Billings, 1995). Athichanagorn et al. (1999) used it to calculate the unknown flow rate history from the long-term PDG pressure data. In this paper, the sliding window technique is applied to analyse the transient pressure data from reservoir systems with nonlinearities.

2.2. Deconvolution

In von Schoroeter et al. method, Eq. (1) is solved not for the unit constant-rate pressure response $p_u(t)$ but for the function $z(\sigma)$, which is based on the derivative of $p_u(t)$ with respect to the natural logarithm of time (von Schroeter et al., 2001, 2004):

$$z(\sigma) = \ln\left[\frac{dp_u(t)}{dln(t)}\right] = \ln\left[\frac{dp_u(\sigma)}{d\sigma}\right] \tag{2}$$

where $\sigma = \ln(t)$. Eq. (2) ensures that $\frac{d\,p_u(t)}{dln(t)}$ is positive. The convolution integral of Eq. (1) becomes a nonlinear convention equation:

$$p(\sigma) = p_0 - \int_{-\infty}^{\ln t} q(t-e^{\sigma})e^{z(\sigma)}d\sigma \tag{3}$$

Considering the error of pressure, rate and regularization curvature constraints, deconvolution is formulated as the unconstrained nonlinear minimization. Based on von Schoroeter et al. method, Levitan (2005) and Levitan et al. (2006) made some improvements. First, the assumption unit-slope in wellbore storage before the first node is removed. The first node t_1 is very small, and