Journal of Natural Gas Science and Engineering 27 (2015) 666–675

Contents lists available at ScienceDirect

Journal of Natural Gas Science and Engineering

journal homepage: www.elsevier.com/locate/jngse

New stress and initiation model of hydraulic fracturing based on nonlinear constitutive equation

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ARTICLE INFO

Article history:
Received 28 March 2015
Received in revised form
31 August 2015
Accepted 3 September 2015
Available online 8 September 2015

Keywords:
Nonlinear behavior of rock
Nonlinear constitutive equation
Hydraulic fracturing
Stress model
Initiation model

ABSTRACT

Many rocks exhibit nonlinear behavior under the effect of internal and external factors; thus, stress and initiation models based on linear elastic theory are not applicable for these rocks. In accordance with the deformation theory of plastic mechanics, a nonlinear constitutive model is developed in this study based on the power-hardening equation using a piecewise approximation method. A new model for the elastic–plastic stress field around the wellbore is then proposed considering the in-situ stress anisotropy. Finally, a new elastic–plastic hydraulic fracturing initiation model is developed, coupled with the maximum tensile strength and Mohr–Coulomb criteria. Calculations and analyses reveal that the nonlinearity of the constitutive relation has significant effects on the stress distribution, initiation mode, and pressure. The plastic yield has little effect on the radial stress but a significant effect on the circumferential stress. When rock yielding occurs, the stress concentration around the wellbore is reduced, and the circumferential stress decreases or cannot be tensile. In this case, the initiation pressure is much higher than that of the linear elastic model, and the initiation mode includes tensile and shear failure. The initiation mode is comprehensively controlled by the in-situ stress, cohesion, tensile strength, power-hardening index, yield stress, and internal friction angle. The initiation orientation of both initiation modes is along the maximal horizontal principal stress direction; however, there is a failure angle for the shear failure. It is more accurate to predict the initiation mode and pressure using the piecewise power-hardening constitutive equation than the Hubbert and Willis model.

1. Introduction

Hydraulic fracturing is considered a successful stimulation technique for increasing the production from conventional low permeability reservoirs and one of the key techniques that render the recovery of tight gas economically viable. Initiation and propagation are two important mechanical issues of hydraulic fracturing (Fig. 1). A fundamental understanding and the prediction of fracture initiation are essential for the efficient and effective design of hydraulic fracturing. The initiation prediction is the primary focus of this study; thus, the propagation after the initiation is not discussed. Since Hubbert and Willis (1957), many other researchers have studied the hydraulic fracturing initiation behavior; however, their studies have mainly focused on hard–brittle rocks (Haimson and Fairhurst, 1967; Hossain et al., 2000; Huang et al., 2012).

With the rapid development of the exploration and exploitation of oil and gas reservoirs, many complex lithology and unconventional reservoirs have received increasing attention (Zou et al., 2013). With the increase of nonbrittle materials, such as clay, the rock characteristics change from elastic–brittle to elastic–plastic (Jarvie et al., 2007). The elastic–brittle rock also transforms into elastic–plastic rock under confining pressure (Paterson and Wong, 2005). Field and laboratory experiments (Papanastasiou et al., 1995; Bohloli and De Pater, 2006) have confirmed that the initiation behavior of elastic–plastic rock is different from that of elastic–brittle rock. Correspondingly, the elastic–plastic rock may fail in tension fracture, and short, branched, and tortuous shear fracture is favorable.

Some works on the analysis of hydraulic fracturing initiation, or borehole stability, have discussed the effect of the plasticity or nonlinearity. The elastic–plastic analysis mainly faces two issues: the stress field around the wellbore based on the constitutive equation and the failure criterion for initiation or collapse.

Most models for stress calculation are based on either nonlinear elasticity or elastoplasticity. In nonlinear elastic models (Santarelli

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http://dx.doi.org/10.1016/j.jngse.2015.09.007
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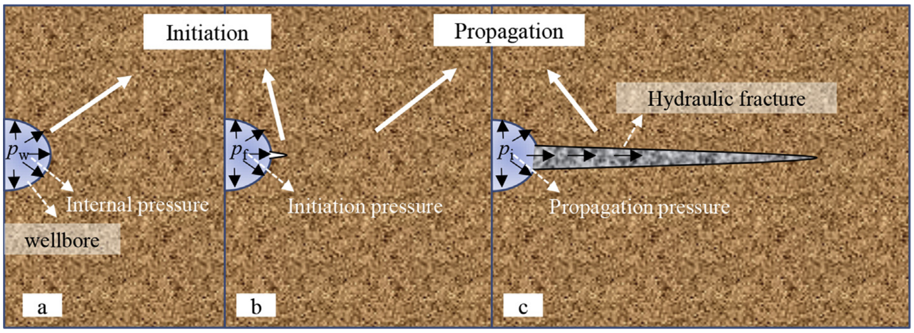

Fig. 1. Sketches of the initiation and propagation of hydraulic fracture.

and Brown, 1987), the stress rate is a linear function of the deformation rate with the Young's modulus being dependent on the hydrostatic pressure. The elastic–perfectly plastic model (Boyce, 1988; Aadnøy and Belayneh, 2004; Xia and Moore, 2006), linear elastic-weakening model (Wang and Dusseault, 1991a, 1991b), and linear hardening model (Morita et al., 1989) cannot sufficiently describe the nonlinearity. Semi-analytical models, such as the nonlinear hardening model, or bifurcation analysis (Bradford and Cook, 1994; Papanastasiou and Cook, 1995, 1999) are inconvenient for application. Numerical models, based on the finite element method, or gradient plasticity theories (Shao and Homand, 1998; Zervos et al., 2002; Xia and Moore, 2006; Roshan and Oeser, 2011) can describe complicated conditions, such as in-situ stress anisotropy, deviation, and the thermo–chemo–poroplastic effect; however, these models are computationally expensive and require significant amounts of input data.

Compared with the wellbore collapse analysis with shear failure, the initiation of hydraulic fracturing is more complex with two failure modes that include tensile and shear failure. Most studies have concluded that application of linear elasticity and the maximum tensile stress–crack criterion under predict the initiation pressure (Wang and Dusseault, 1991a, 1991b; Xia and Moore, 2006; Bohloli and De Pater, 2006). However, some research studies have demonstrated that the fracturing pressure may be lower than in the cases where the linear elastic model is used (Papanastasiou and Cook, 1995). Aadnøy and Belayneh (2004) also developed an elasto–plastic model by considering only tensile failure in the initial borehole fracturing process. No unified conclusion has been reached regarding the initial mode and pressure of hydraulic fracturing.

In this study, a new analytical hydraulic fracturing initiation model is developed based on a nonlinear constitutive equation. In accordance with the deformation theory of plasticity, a power-hardening constitutive equation is used based on the method of piecewise approximation. The wellbore stress distribution model is then built based on the nonlinear constitutive equation. The initiation model is developed coupled with the maximum tensile strength and Mohr–Coulomb criteria. The effect of the constitutive parameters on the stress distribution, initial modes, and initial pressure are all analyzed in the study. The effect of the in-situ stress anisotropy is also discussed.

2. Materials and methods

2.1. Stress–strain relationship

In general, rock behavior is divided into brittle and ductile behavior from the post-failure curve of the stress–strain, as shown in Fig. 2 (Jin et al., 2014). The behavior of the traditional hydraulic fracturing pay-zones, such as hard sandstone, are more likely to resemble behavior A. Therefore, in this case, satisfactory results can be obtained with linear elasticity in the initiation pressure analysis. However, for behavior A, this approach ignores the nonlinear part before the peak. Behavior C is more common in the field application. In this type of failure, even though the rock fails immediately when the peak stress reaches the rock strength, there is an obvious nonlinear stage between the linear and post-failure stages. This nonlinear stage should be considered in the constitutive equation, and these types of rocks are the target of this study.

Laboratory and theoretical studies (Liu et al., 2009; Amann et al., 2011, 2012) have revealed that the damage process of brittle rocks includes five stages: crack closure (Stage I), a linear elastic region (Stage II), stable crack growth (Stage III), unstable crack propagation (Stage IV), and macrocrack formation (Stage V). As illustrated in Fig. 3, σ_{cc} is the creak closure stress, σ_{ci} is the crack initiation stress, σ_{cd} is the crack damage stress level, and σ_c is the peak stress. The crack closure stage, which can be attributed to the closure of the existing microcracks, is negligible for the low-permeability sandstone and tight shale. The crack initiation and crack damage are used to describe the onset of damage (initiation) and crack coalescence to form macroscopic fractures, respectively.

From the perspective of the macroscopic stress–strain relationship, the linear elastic behavior is followed by nonlinear behavior that includes stable and unstable crack growth stages. It has also been demonstrated that the initial yield point (crack initiation) is the dividing point between the linear elastic process and plastic process (nonlinear behavior) and that the failure point is the final result of the plastic process (Gao et al., 2006). Experiments and numerical simulation (Diederichs et al., 2004; Liu et al., 2009; Amann et al., 2011) have shown that the crack initiation stress coincides with the point where the stress–volumetric strain and the stress–lateral strain curves depart from linearity. The crack initiation stress is also equivalent to the acoustic emission (AE) initiation point (Martin, 1997; Cai et al., 2004).

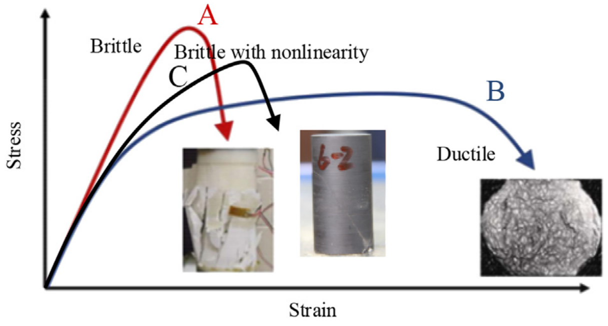

Fig. 2. Brittle with nonlinear behavior compared with brittle and ductile stress–strain diagrams (Jin et al., 2014).