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Wellbore stability analysis based on a new strength criterion

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ABSTRACT

Wellbore stability is critical to the overall safety of drilling. The adoption of a better strength criterion greatly improves the accuracy of wellbore stability prediction. Although the Mohr–Coulomb strength criterion is widely used for wellbore stability analysis, it is difficult to use its linear form to accurately describe the nonlinear characteristics of the rock strength under different confining pressures. In the present study, we developed a new criterion, which we refer to as the single-parameter parabolic criterion, for wellbore stability analysis based on the Mohr strength criterion with a parabolic failure envelope. The fitting precisions of the Mohr–Coulomb criterion, the Hoek–Brown criterion, and the developed single-parameter parabolic criterion were statistically analyzed. It was found that the single-parameter parabolic criterion accurately describes the relation between the rock strength and confining pressure. Based on the single-parameter parabolic strength criterion, a collapse pressure calculation model for vertical and inclined wells was developed and applied to a field case, and the new criterion was further demonstrated to be superior to the Mohr–Coulomb criterion.

1. Introduction

The calculation accuracy of the collapse pressure is directly determined by the rock strength criterion, and an optimized strength criterion improves the precision in predicting the wellbore stability (Ewy, 1999; Islam et al., 2010; Al-Ajmi and Zimmerman, 2006a, 2006b; Al-Ajmi, 2006; Maleki et al., 2014). The strength criteria that are most commonly used for wellbore stability analysis are the Mohr–Coulomb criterion and the Drucker–Prager criterion (Ewy, 1999; Al-Ajmi and Zimmerman, 2006a; McLean and Addis, 1990a; McLean and Addis, 1990b). Recently, Al-Ajmi and Zimmerman proposed the use of the 3D Mogi–Coulomb strength criterion for brittle rock wellbore stability analysis (Al-Ajmi and Zimmerman, 2006a, 2005, 2009; Al-Ajmi, 2006). It has also been shown by substantial experimental data (Mogi, 2007; Yang et al., 2013; Singh et al., 2011; Singh and Singh, 2012; Yu et al., 2009; Mahtab and Goodman, 1968) that the rock strength and the confining pressure are not linearly correlated. Thus, the linear Mohr–Coulomb strength criterion does not accurately describe the non-linear strength characteristics of rocks. Al-Ajmi, Zimmerman, and Ewy (Ewy, 1999; Al-Ajmi and Zimmerman, 2006a, 2006b, 2005) concluded that the Mohr–Coulomb criterion was too conservative for predicting the collapse pressure and that the Drucker–Prager criterion underestimated the collapse pressure. Incidentally, the prediction of the collapse pressure should be as accurate as possible because a high prediction suggests a greater likelihood of the formation's being crushed, leading to undue degradation of the wellbore stability (Edwards et al., 2004; McLellan and Cormier, 1996; Santarelli et al., 1992), whereas a low prediction suggests the likelihood of formation collapse. It is crucial to maintain the stability of a borehole, which is primarily achieved using a reasonable configuration of the drilling fluid density and the well trajectory (Al-Ajmi and Zimmerman, 2006a, 2009; Morita, 2004; Zhou et al., 1996; Ma and Chen, 2015) based on the establishment of an accurate collapse pressure profile along the wellbore extension using a reasonable rock strength criterion.

Using a statistical index that describes the correlation between the experimental data and theoretical value, we analyzed the fitting accuracies of the Mohr–Coulomb criterion, the Hoek–Brown criterion, and the newly developed single-parameter parabolic criterion to investigate the relative accuracy of the new criterion for evaluating the rock strength under different confining pressures. We further used the new criterion to develop a model for predicting the collapse pressure of vertical and inclined wells and demonstrated that the collapse pressure model was more suitable for the wellbore stability analysis compared to the Mohr–Coulomb

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criterion through a case application.

2. Strength criterion

A rock strength criterion can be expressed in two forms, namely, the implicit shear stress and normal stress expression ($c\tau = g(\sigma)$) and the explicit principal stress expression ($\sigma_1 = f(\sigma_3)$). However, the mechanical meaning of the implicit expression has not been thoroughly elucidated, and the parameters are relatively difficult to determine (You, 2010a). Therefore, this paper primarily discusses the explicit strength criterion as expressed in terms of the principal stress.

2.1. Mohr-Coulomb criterion

The implicit stress can be expressed as follows (Coulomb (1776)):

$$|\tau| = c + \sigma_n \tan\varphi \tag{1}$$

By coordinate transformation, equation (1) can be expressed in the form of the explicit principal stress:

$$\sigma_1 = \frac{2c\cos\varphi}{1-\sin\varphi} + \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3 \tag{2}$$

Equation (2) can also be written in terms of the maximum shear stress τ_{max} and the mean maximum principal stress $\sigma_{m,2}$ (Jaeger and Cook, 1976):

$$\tau_{\max} = c\cos\varphi + \sin\varphi\sigma_{m,2} \tag{3}$$

where $\tau_{max} = (\sigma_1 - \sigma_3)/2$ and $\sigma_{m,2} = (\sigma_1 + \sigma_3)/2$.

2.2. Hoek–Brown criterion

The Hoek–Brown criterion has the original form and the generalized form. The original empirical Hoek–Brown criterion was established in 1980 by E. Hoek and E. T. Brown based on the study of the brittle failure of undisturbed rocks and the deformation model of fractured rock mass (Hoek and Brown, 1980):

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \tag{4}$$

Considering the limitations and deficiencies of this original criterion, in 1995, Hoek (Hoek et al., 2005) proposed the generalized empirical Hoek–Brown criterion, and the expression is suitable for various types of rock:

$$\sigma_1 = \sigma_3 + \sigma_c\left(m\frac{\sigma_3}{\sigma_c} + s\right)^a \tag{5}$$

where a is a material-related constant. Equation (5) reduces to the original Hoek–Brown criterion for $a = 0.5$.

2.3. Single-parameter parabolic criterion

What we refer to as the single-parameter parabolic criterion was originally developed from the Mohr strength criterion with a parabolic failure envelope. This criterion can be expressed as follows (Li et al., 2010; Pengnian, 1979):

$$\sigma = \frac{\tau^2}{aT_0} - T_0 \tag{6}$$

The normal and shear stresses at different inclined angles (θ) in a rock sample respectively satisfy the following equations:

$$\begin{cases} \sigma = \sigma_1\cos^2\theta + \sigma_3\sin^2\theta = \frac{1}{2}(\sigma_1+\sigma_3) + \frac{1}{2}(\sigma_1-\sigma_3)\cos 2\theta \\ \tau = (\sigma_1-\sigma_3)\sin\theta\cos\theta = \frac{1}{2}(\sigma_1-\sigma_3)\sin 2\theta \end{cases} \tag{7}$$

The Mohr strength criterion in equation (6) and the Mohr stress circles are depicted in Fig. 1.

Here, β is the angle between the σ axis and the tangent of the rock failure envelope (implicit strength criterion) at (σ, τ), and θ is the rock failure angle (i.e., the angle between the failure surface and the maximum principal stress plane at failure). The relationship between β and θ is expressed by equation (8):

$$\beta + 90^\circ = 2\theta, \quad \tan\beta = d\tau/d\sigma \tag{8}$$

The simplified explicit expression of the Mohr strength criterion with a parabolic failure envelope on the principal stress plane can be obtained by substituting equations (7) and (8) into equation (6):

$$(\sigma_1 - \sigma_3)^2 = 2aT_0(\sigma_1+\sigma_3) + 4aT_0^2 - a^2T_0^2 \tag{9}$$

If we let $A = 2aT_0$ and $C = 4aT_0^2 - a^2T^2$, equation (9) can be rewritten as

$$(\sigma_1-\sigma_3)^2 = A(\sigma_1+\sigma_3) + C \tag{10}$$

Simultaneously, the maximum principal stress is equal to the uniaxial compressive strength when the confining pressure is zero. Hence, it is necessary for equation (10) to satisfy the constraint

$$\sigma_1|_{\sigma_3=0} = \sigma_c \tag{11}$$

According to a fitting analysis, conducted by Mingqing Y (You, 2010a), of seven groups of rock strength data obtained from 6 different types of rock, the parameter A exhibits a good linear fitting performance with the uniaxial compressive strength σ_c. Therefore, we can set A $= 2(\sigma_c - \sigma_d)$ to further determine the parameter A, where σ_d is a constant that is unrelated to σ_c. Furthermore, by substituting equation (11) into equation (10) and letting A $= 2(\sigma_c - \sigma_d)$, we obtain

$$(\sigma_1-\sigma_3)^2 = 2(\sigma_c-\sigma_d)(\sigma_1+\sigma_3) + \sigma_c^2 - 2\sigma_c(\sigma_c-\sigma_d) \tag{12}$$

Equation (12) can be simplified as

$$\sigma_1 = \sigma_3 + \sigma_c - \sigma_d + 2\sqrt{(\sigma_c-\sigma_d)\sigma_3 + \sigma_d^2} \tag{13}$$

Utilizing the explicit parabolic Mohr strength criterion in equation (13), Mingqing Y (You, 2010a) obtained the strength

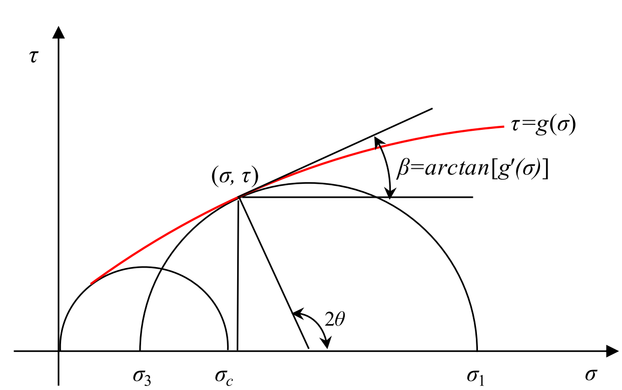

Fig. 1. Mohr strength criterion and stress circles.