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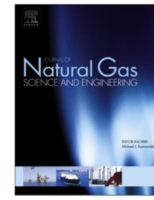

A novel binomial deliverability equation for fractured gas well considering non-Darcy effects

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ABSTRACT

Hydraulic fracturing is one of the most effective methods to improve the productivity of low permeability reservoirs. Now, the effects of non-Darcy flow mainly caused by high velocity gas in hydraulic fracture are paid increasing attention to. But the binomial deliverability equation influenced by non-Darcy effects has never been seen in any paper for the difficulty in coupling linear flow model in the fracture and radial flow model in the reservoir.

Based on equivalent radius models for hydraulic fracture, the authors put forward the concept of **CTER**, which is short for **conductive thickness between equivalent radii**. The value of equivalent radius for finite conductivity fracture is smaller than that of infinite conductivity fracture because of the existence of fracture conductivity. CTER whose permeability is equal to reservoir permeability is a theoretical radial conductive thickness existing between equivalent radius models for finite conductivity fracture and infinite conductivity fracture. By making use of CTER, we deduce the novel binomial deliverability equation with respect to the inertial non-Darcy effects in the fracture. With the help of the new concept and the new equation, it is found that non-Darcy effect caused by high flow rate of gas in the fracture influences high permeability reservoirs (>5 mD) more dramatically than those with low permeability (<1 mD).

1. Introduction

In 1856, Henry Darcy developed his famous flow correlation equation named Darcy's Law through sand pack configurations (Miskimins, 2005). Forty-five years later, Forchheimer (1901) observed Darcy's equation deviated from linearity when fluid flowed at increased rates. The deviation caused by high velocity of flowing fluid is known as non-Darcy effect. With the consideration of non-Darcy effect near wellbore, Houpeurt (1959) developed a binominal gas-flow equation which is widely used in gas deliverability tests such as flow after flow test, isochronal test, modified isochronal test and so on.

Massive hydraulic fracturing treatments are considered as excellent stimulation means of boosting the productivity of both damaged wells and wells producing from low permeability reservoirs. Studies on vertically fractured wells have achieved glorious achievements since the pioneering work of Gringarten et al. (1975) who made an extraordinary contribution to the development of transient pressure data analysis and type curve analysis for infinite conductivity fracture as well as the work of Cinco-Ley et al. (1978) and Cinco-Ley and Samaniego (1981) who presented semi-analytical and analytical solutions to finite conductivity fracture. Expressing the reservoir equation in Elliptic Coordinates system and representing the fracture as a degenerate ellipse, Riley et al. (1991) developed exact analytical solutions for the pressure response of a vertical finite conductivity fracture of elliptical cross-section. In his paper, he proposed an equivalent radius for elliptical fracture. Equivalent radius model was first developed to solve skin damage problems of vertical well. Raghavan and Joshi (1993) and Raghavan et al. (1997) deduced an equivalent wellbore radius model for the vertical hydraulic fracture which was rectangular and infinite conductivity. Based on the work of Riley et al. (1991) and by solving the integral equation and asymptotic analyses, Wang et al. (2004) presented a general productivity formula for vertical rectangular fracture with finite conductivity and defined a new equivalent radius which was influenced by fracture half-length and dimensionless fracture conductivity. As was shown in his paper, the new equivalent radius model matched with the model proposed by Prats et al. (1960) very well. Then, Wang and Li (2010) used the new equivalent radius model for rectangular fracture to compute the

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production of fractured horizontal well. Recently, Wang et al. (2013) and Wang and Wang (2014) achieved the solution for asymmetrically fractured wells in Laplace domain which could easily consider effects of wellbore storage and fracture skin damage.

In recent years, non-Darcy flow in both artificial and natural fractures has been paid increasing attention to. For example, one simple reason for this increased interest is that non-Darcy effects can significantly decrease well productivity. Smith et al. (2004) found that non-Darcy effects accounted for a 35% decrease in productivity in a fractured high rate oil well and also showed a productivity index reduction of ~20% in a high rate (120MMCFD) gas well.

The effects of non-Darcy flow specifically in hydraulic fracturing operations were first addressed by Cooke (1973). Since then much work has been covered to address non-Darcy effects. Holditch and Morse (1976) used numerical simulation model to study the effect of turbulent flow in both reservoir and fracture. Guppy et al. (1981, 1982) presented comprehensive numerical and semi-analytical models for non-Darcy flow in finite-conductivity fractured wells. The models are capable of integrating fracture and reservoir properties. Vincent et al. (1999) showed non-Darcy effects were significant even in wells whose rate were considered to be low in current industry standards. Amini and Peter (2010) analyzed the problem of a horizontal well with multiple transverse fractures of unknown number and size, where the non-Darcy effects both in the fracture and the additional inflow into the well were considered simultaneously. This is achieved by the application of the distributed volumetric sources (DVS) method (Amini, 2007), allowing fast calculation of production performance. Song et al. (2012) constructed a model for calculating critical production rates of bottom water coning well with the consideration of non-Darcy flow around the well.

Overall, many researchers have emphasized the importance of non-Darcy effects on gas flow performance in hydraulic fracture and achieved many distinguished results, but an effective production model and its relevant analytical solution to steady or pseudo steady state have not been addressed in any paper. Because it is difficult to get an analytical settlement after coupling reservoir model and fracture model when taking the non-Darcy effects into consideration. As a result, traditional binomial deliverability equation proposed by Houpeurt (1959) could not be used in the fractured gas well deliverability tests any more.

The objective of this paper is to present a novel analytical equation in the form of binomial deliverability equation for fractured wells considering non-Darcy flow in the fracture. The authors introduce a new concept named CTER, which is short for conductive thickness between equivalent radii. With the help of CTER, we could deduce the novel binominal deliverability equation which could easily: (1) calculate fractured gas well production, (2) use techniques of gas well test to analyze field production data of fractured wells and further to get relevant reservoir parameters, (3) evaluate the effects of non-Darcy flow in the fracture.

2. Theoretical bases

2.1. Binomial deliverability equation

In 1856, Henry Darcy developed his famous Darcy's law describing laminar flow through porous media. As is shown in Eq. (1), the pressure gradient is directly proportional to fluid velocity.

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mu}{k}v \tag{1}$$

Forty-five years later, Forchheimer observed deviation from the linearity of Darcy's equation at increased flow rates (Miskimins, 2005). The additional pressure drop was caused by the frequent acceleration and deceleration of the moving fluid particles. Forchheimer (1901) proposed a second proportionality constant that would account for this non-linearity. The new equation could describe the high velocity fluid flow considering inertial effect very well:

$$\frac{\mathrm{d}P}{\mathrm{d}x} = av + bv^2 \tag{2}$$

Cornel and Katz (1953) described these inertial effects with a more detailed equation (3).

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mu}{k}v + \beta\rho v^2 \tag{3}$$

Comparing Eq. (2) with Eq. (3), we get that $a = \mu/k$, $b = \beta\rho$

A binomial deliverability equation developed by Houpeurt (1959) is widely used for vertical well when inertial effects exist around the wellbore.

$$P_e^2 - P_{wf}^2 = \Delta P_D^2 + \Delta P_{nD}^2 = Aq_{sc} + Bq_{sc}^2 \tag{4}$$

$$\text{Where,}\quad A = \frac{1.291\times10^{-3}T\overline{\mu}\overline{Z}}{k_r h}\left(\ln\frac{r_e}{r_w} + S\right);$$
$$B = 2.828\times10^{-21}\frac{\beta\gamma_g\overline{Z}T}{h^2}\left(\frac{1}{r_w} - \frac{1}{r_e}\right)$$

Rearranging equation (4), we get:

$$\frac{P_e^2 - P_{wf}^2}{q_{sc}} = A + Bq_{sc} \tag{5}$$

Using $P_e^2 - P_{wf}^2/q_{sc}$ as y-axis and using q_{sc} as x-axis, if we draw real production data on Cartesian Coordinates system, we will obtain a straight line with a slope B and an intercept A. Once A and B are obtained, reservoir parameters and inertial factor β could be calculated. The matching technique related above is called Houpeurt technique. The main purpose of this paper is to develop a new Houpeurt equation for fractured gas wells considering non-Darcy flow in the fracture with the help of equivalent radius theories.

2.2. Theory of fracture conductivity

The fracture conductivity can be defined quantitatively with the hydraulic fracture conductivity C_f, which is a multiplication product of fracture width (w) and fracture permeability (k_f), given by Prats et al. (1960) as below:

$$C_f = k_f w \tag{6}$$

Comparing with fracture conductivity C_f, the dimensionless conductivity C_{fD} which is widely used in fractured well test analysis (Cinco-Ley et al., 1978; Cinco-Ley and Samaniego, 1981) has more application value:

$$C_{fD} = \frac{k_f \cdot w}{k_r \cdot x_f} \tag{7}$$

2.3. Infinite conductivity fracture equivalent radius model

In order to analyze the productivity of fractured horizontal wells, Raghavan and Joshi (1993) deduced an equivalent wellbore radius model for vertical hydraulic fracture and used superposition principle to compute the conductivity of fractured horizontal well. Combining his innovative ideas with the achievement of Cinco-Ley