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Productivity equation of fractured well in CBM reservoirs

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A B S T R A C T

Due to complex process of CBM transport associated with desorption and diffusion, there has not been an explicit and accurate prediction formula of CBM production for fractured wells. This article presented a productivity equation of fractured well in CBM reservoir regarding desorption and diffusion. Elliptical flow pattern exists around the hydraulically fractured well, and the flow field was divided into two regions. One is high-velocity non-linear flow in artificial fracture, and the other is Darcy flow in elliptical region controlled by artificial fracture. Mathematical models for the elliptical gas flow were established based on conservation of mass and momentum equations, in which Langmuir equation, Fick pseudo-steady state law, and function of pseudo pressure were combined to consider effect of desorption and diffusion. The productivity equation of hydraulically fractured well was presented by coupling the analytical solutions to elliptical gas flow. Effect on gas rate of reservoir properties and production parameters such as desorption rate, diffusion coefficient, drawdown pressure, half-length of artificial fracture, and flow conductivity were clarified based on the productivity equation. It is seen that there exist optimal drawdown pressure and optimal half-length of hydraulic fracture dependent on other parameters. In the excess of optimal values, gas rate will reach a plateau and respond with little increment along with the increase of drawdown pressure or half-length. Consequently, the research provides direct insight of the effect of various parameters on gas rate and theoretical foundation for optimization design of CBM development.

1. Introduction

Productivity equation provides an effective tool and direct insight for gas reservoir engineers to predict production capacity and optimize reservoir design (Rahman et al., 2007; Zhu et al., 2011). However, there has been lack of analytical models for productivity due to complex transport mechanism of CBM including desorption, diffusion and seepage. CBM reservoir is dual porous media (Singh, 2011), constituted by micro-fracture network and matrix. CBM desorbs from matrix and diffuses into micro-fractures, where seepage is the main transport mechanism. CBM reservoirs in China feature low-permeability reservoir properties, leading to wide application of hydraulic fracturing technique for enhancement of CBM production. There exist urgent needs of theoretical analysis of optimal design of hydraulic fracturing.

Researchers have done a lot of work on evolution of coal permeability (Wu et al., 2010; Wang et al., 2011; Tao et al., 2012; Pan and Connell, 2012) and modeling for effect of various properties on CBM production (Nie et al., 2012; Adeboye and Bustin, 2011; Wang et al., 2012; Karacan and Okandan, 2000; Anna, 2003; Connell, 2009; Aminian and Ameri, 2009; Wang and Zhang, 2010). In general, numerical simulation, which is complex and time-consuming, is adopted to predict productivity of CBM well.

The objective of the study is to present an analytical productivity equation of a single vertical well with a hydraulic fracture, considering desorption and diffusion. It is assumed that elliptical flow regime dominates flow performance in low-permeability dual-porosity CBM reservoirs. The flow field around the hydraulic fracture was divided into two regions. One is high-velocity non-linear flow in artificial fracture, and the other is Darcy flow in elliptic region controlled by artificial fracture. Based on Langmuir equation and pseudo-steady diffusion in matrix and Darcy flow in the network of micro-fractures, mathematical models for methane flow in the above two regions were established. Analytical solutions were presented and productivity equation was derived.

2. Description of physical model

Due to poor physical properties of CBM reservoirs, hydraulic fracturing is frequently adopted for enhancement of gas recovery.

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Roberts (1981) and Thompson (1981) separately considered the flow behavior of a vertical fractured well in tight gas reservoir, and came to a conclusion that elliptical flow plays a major role in the performance of fractured tight gas wells from the perspective of analyzing production data. Riley (1991) and Liu (1987) proposed mathematical models for elliptical flow behavior of a vertical fractured well. These perspectives lead us to the assumption that an elliptical flow model is representative and validated for fractured wells in tight gas reservoirs such as low-permeability CBM reservoirs. Therefore, the flow field around artificial fractures could be divided into two regions (Liu, 1987; Wang et al., 2004), as depicted in Fig. 1. One is high-velocity non-linear flow in artificial fracture, and the other is Darcy flow in elliptic region controlled by artificial fracture.

For simplification the following assumptions are put forward. Most of CBM is absorbed in the matrix (Harpalani and Chen, 1997), and water only exists in micro-fractures. Average pressure of the reservoir system is regarded as the pressure of CBM in the matrix. Temperature in the coal bed remains constant. Absorption and desorption accord with Langmuir equation (King and Ertekin, 1988). Desorption process and diffusion process are considered. Flow in micro-fractures is considered to follow Darcy's law.

3. Mathematical models and productivity equation of fractured well

3.1. Model for flow in elliptic region and its analytical solution

3.1.1. Continuity equation associated with CBM desorption

Euler method is adopted to describe the continuous medium model, and each infinitesimal cube includes micro-fractures and matrix, as shown in Fig. 2. Methane flow in coal bed is subject to three processes: gas desorption from coal grains, gas diffusion in matrix and flow in network of micro-fractures. Consider a control volume having the shape of a rectangular parallel-piped box of dimensions dx,dy,dz centered at the point $M(x, y, z)$ inside the flow domain. Methane enters and leaves the box through its surfaces, desorbs from matrix and diffuses into micro-fractures in the box simultaneously. A statement of conservation for the mass of methane entering, leaving, desorbing from coal and being stored in the box was derived as follows.

As is known, the total excess of mass inflow over outflow per unit volume of porous media during dt was obtained:

$$-\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}+\frac{\partial(\rho v_z)}{\partial z}\right) = -\nabla\cdot(\rho\vec{v}) \tag{1}$$

During the CBM development, the methane continuously desorbed from matrix and diffused into micro-fractures, serving as the source term q_d. The source term was presented as the mass quantity of methane diffusing into mirco-fractures per unit volume and per unit time.q_d could be defined as follows:

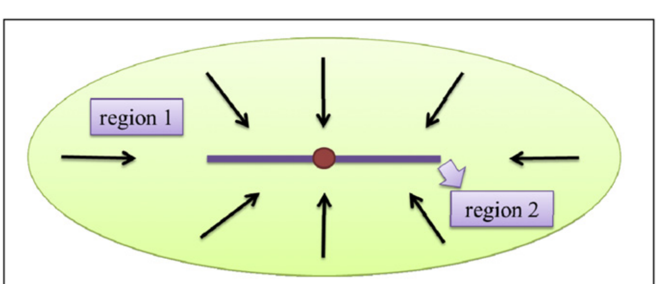

Fig. 1. Schematic diagram of elliptical flow field around hydraulic fractures.

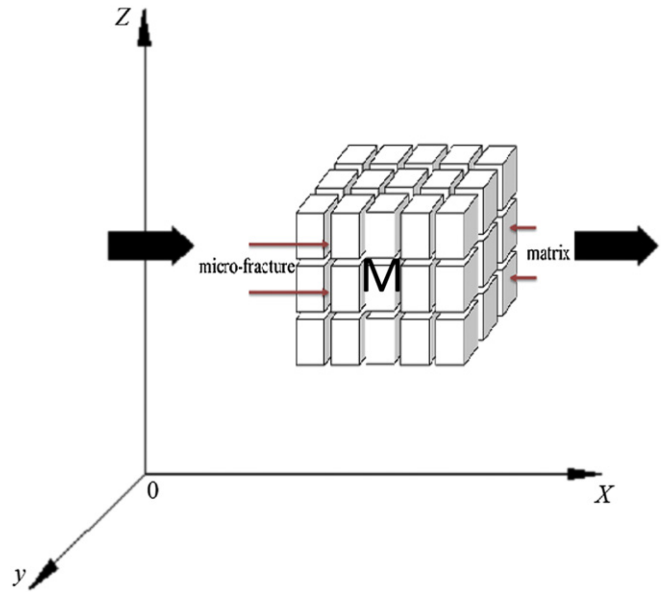

Fig. 2. Schematic illustration of continuity equation in double media system.

$$q_d = -\omega\frac{dc_m}{dt} \tag{2}$$

Assuming that the methane diffusion follows Fick's first law,

$$\frac{dc_m}{dt} = D_m F_s(c_m - c_2) \tag{3}$$

where $c_m - c_2 = Mp_m\phi/RTZ$.

In a saturated porous medium domain, the mass of methane present in the cube is expressed as $\rho\phi dxdydz$. The rate of change of the mass per unit volume per unit time is given by $\partial/\partial t(\rho_g\phi)$.

Therefore, the continuity equation was established:

$$\frac{\partial}{\partial t}\left(\rho_g\phi\right)+\nabla\cdot\left(\rho_g\vec{v}\right)+q_d = 0 \tag{4}$$

3.1.2. Governing equation

Motion equation:

$$\vec{v} = -\frac{k}{\mu}\nabla p \tag{5}$$

Gas density equation is presented as:

$$\rho_g = \frac{T_{sc}Z_{sc}\rho_{gsc}}{p_{sc}}\cdot\frac{p}{TZ} \tag{6}$$

The isothermal gas compressibility expression is:

$$C_\rho = \frac{-\frac{dV}{V}}{dp} = -\frac{1}{V}\cdot\frac{dV}{dp} = \frac{1}{p}-\frac{1}{Z}\cdot\frac{dZ}{dp} \tag{7}$$

Combining Eqs. (5)–(7), the following equation can be established after simplification and derivation:

$$\frac{\partial\left(\rho_g\phi\right)}{\partial t} = \frac{T_{sc}Z_{sc}\rho_{gsc}\phi}{p_{sc}T}\left[\frac{1}{p}-\frac{1}{Z(p)}\cdot\frac{\partial Z(p)}{\partial p}\right]\cdot\frac{p}{Z(p)}\frac{\partial p}{\partial t} \tag{8}$$