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Short communication

Wave steering effects in anisotropic composite structures: Direct calculation of the energy skew angle through a finite element scheme

D. Chronopoulos

Institute for Aerospace Technology & The Composites Group, The University of Nottingham, NG7 2RD, UK

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ABSTRACT

A systematic expression quantifying the wave energy skewing phenomenon as a function of the mechanical characteristics of a non-isotropic structure is derived in this study. A structure of arbitrary anisotropy, layering and geometric complexity is modelled through Finite Elements (FEs) coupled to a periodic structure wave scheme. A generic approach for efficiently computing the angular sensitivity of the wave slowness for each wave type, direction and frequency is presented. The approach does not involve any finite differentiation scheme and is therefore computationally efficient and not prone to the associated numerical errors.

1. Introduction

Understanding complex wave phenomena is of paramount importance for the successful application of ultrasonic techniques within the non-destructive testing (NDT) and biomedical fields. Accurate and efficient modelling of elastic wave propagation complex phenomena in composite structures play a crucial role in the development of robust algorithms for damage detection and localization. One of the most prominent of these phenomena is the so-called energy skewing (see Fig. 1), induced by the angular divergence between the phase and group velocities for non-isotropic configurations. Wave skewing results in a non-uniform distribution of energy along the wavefront. An inaccurate description of the skewing effect in the computational models and NDT algorithms can well result in an incorrect prediction of damage location [1,2] and type.

Directional dependence of the wave slowness characteristics in non-isotropic structures has been well discussed and investigated by several researchers. In [3] the authors demonstrated a material anisotropy-based, beam-steering scheme for electronically steering an acoustic beam over an angle larger than 70° in a TeO_2 crystal. The idea was based on the pronounced angular dependency of the wave skewing angle in the same material. Wave beam steering through the employment of phased array transducers [4] has been discussed within the context of several applications including biomedical imaging [5], structural health monitoring [6–8] and acoustic applications [9]. With regard to layered cellular composites, the researchers in [10–12] derived wave propagation models based on Bloch's theorem in order to show how band-gaps and strong acoustic focusing can be affected by structural anisotropy in periodic lattice structures.

Calculation of the wavefront curve has formed the basis for most researchers in order to quantify wave steering effects. The wave skewing angle has been calculated by a number of authors through a variety of approaches, including the application of a Fresnel approximation to the wave propagation problem [13], derivation through the propagating group velocities in two orthogonal directions within the panel [14], as well as through a Finite Differentiation (FD) approach [15]. To the best of the author's knowledge, there is currently no expression directly quantifying the wave skewing effect as a function of the mechanical characteristics of the non-isotropic structure.

The principal objective and contributing novelty of this study is the derivation of a systematic and robust expression relating the wave energy skew angle to the material characteristics of the composite structure under investigation. A robust FE-based approach for efficiently computing the angular sensitivity of the wave phase velocities for each wave type, direction and frequency is presented. The considered structure can be of arbitrary layering and material characteristics as FE modelling is employed. The exhibited scheme is able to compute the wavenumber angular sensitivity (and subsequently the energy skew angle) by determining and post-processing a single solution of the system. This overcomes the drawbacks of the currently employed FD approaches.

E-mail address: Dimitrios.Chronopoulos@nottingham.ac.uk

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Nomenclature

$\mathbf{B}$	shape function derivative matrix of a single FE	c_g	group velocity
$\mathbf{C}_0$	elastic stiffness matrix at the material principal axis	k	wavenumber
$\mathbf{J}$	Jacobian matrix of a single FE	l_x, l_y, l_z	dimensions of a single FE
$\mathcal{K}$	intermediate stiffness matrix employed for the assembly of $\mathbb{K}$	s	wave slowness
$\mathbb{M}, \mathbb{K}$	mass and stiffness matrices of the periodic element	w	wave type index
$\mathbf{R}$	displacement phase transformation matrix	$\mathbf{x}$	wave mode shape vector for the elastic waveguide
$\mathbf{T}$	coordinate transformation matrix	ε	propagation constant
$\mathbf{k}$	stiffness matrix of a single FE	θ	wave propagation angle
$\mathbf{q}$	physical displacement vector for the elastic waveguide	η, ξ, μ	local FE coordinates
L_x, L_y	dimensions of the modelled periodic segment	λ	eigenvalue of the wave propagation eigenproblem
L, R, B, T, I	left, right, bottom, top sides and interior indices	ψ	energy skew angle
N	number of elements	ξ	coordinate transformation angle
		ω	angular frequency

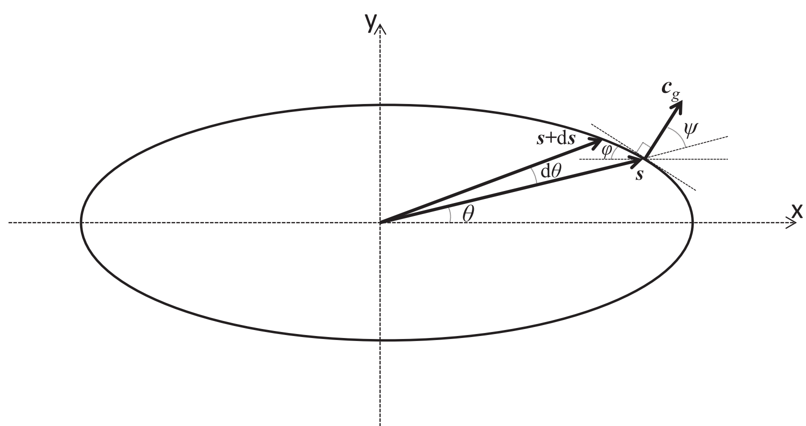

Fig. 1. Illustration of the group velocity being perpendicular to the wave slowness curve for a non-isotropic structure. A wave energy skew angle ψ is thus formed. An infinitesimal change of angle dθ and slowness ds is also shown. The angle ϕ is formed between the horizontal and the tangent.

The paper is organized as follows: In Section 2 a general expression is derived for the angle of the propagating energy wavefront as well as the skew angle between the phase and group velocities for each wave type as a function of the wavenumber angular sensitivity. In Section 3 a direct expression of the wavenumber sensitivity with respect to the direction of propagation is derived within a FE modelling context. Numerical case studies validating the computational scheme are presented in Section 4. Conclusions on the exhibited work are eventually drawn in Section 5.

2. Calculation of the wave energy skew angle

Slowness curves are particularly useful for visualizing the direction of the group velocity (see Fig. 1). On the other hand, the velocity of the wavefront (defined as the locus of ray velocity vectors along all directions starting from the origin) in the direction normal to the wavefront is known as the phase velocity. In an anisotropic material, the phase and group velocities are generally different [16] and a clear distinction between the two should be made to ensure that the correct velocity profile is employed when performing health monitoring with an ultrasonic device. The physical difference between the phase and group velocities can be described by considering a propagating wave packet (see Fig. 1). The wavefronts remain normal to the phase velocity direction θ (or equivalently, parallel to the transducer surface exciting the packet), however due to material anisotropy the wave packet skews away from the normal direction by an angle ψ and instead travels along a shifted ray path. The velocity of the wave packet envelope is given by the group velocity c_g. It has been well documented [14] that the group velocity vector is always perpendicular to the tangent of the slowness curve. Moreover, it is reminded that the slowness of a wave w can be expressed as $s_w = \frac{k_w}{\omega_w}$.

When the angular rate of change for each propagating wavenumber k_w is known (see Section 3), the skew angle ψ_w can be determined through geometric considerations. In Fig. 1, a representation of an infinitesimal change of angle dθ and correspondingly of slowness ds_w is drawn. In the same figure the angle of the tangent to the slowness with respect to the horizontal ϕ is shown. As vector c_g is perpendicular to the drawn tangent and s_w forms an angle θ to the horizontal, the skew angle ψ_w can be determined as

$$\psi_w = \frac{\pi}{2} - \theta - \phi_w \quad 0 \leqslant \theta < \pi \tag{1a}$$

$$\psi_w = \frac{3\pi}{2} - \theta - \phi_w \quad \pi \leqslant \theta < 2\pi \tag{1b}$$

It is straightforward to deduce that

$$\tan(\phi) = \frac{(s + \mathrm{d}s)\sin(\theta + \mathrm{d}\theta) - s\sin\theta}{s\cos\theta - (s + \mathrm{d}s)\cos(\theta + \mathrm{d}\theta)} = \frac{(k + \mathrm{d}k)\sin(\theta + \mathrm{d}\theta) - k\sin\theta}{k\cos\theta - (k + \mathrm{d}k)\cos(\theta + \mathrm{d}\theta)} \tag{2}$$