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Short communication

A two-dimensional model on the coupling thickness-shear vibrations of a quartz crystal resonator loaded by an array spherical-cap viscoelastic material units

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ABSTRACT

We establish a two-dimensional model on the coupling thickness-shear mode (TSM) vibrations of a quartz crystal resonator (QCR) carrying an array of spherical-cap (SC) viscoelastic material units. The electrical admittance of the compound QCR system is described directly in terms of the physical properties of the surface material units. The admittance spectra about the tendon stem cells (TSCs) acquired from our calculation are compared with the existing experiment data and found to be consistent with each other, indicating our model has good veracity and reliability in analyzing the mechanical properties of covered loadings. Furthermore, we calculate admittance spectra of surface Epoxy Resin (SU-8) units with different geometrical configurations and bulk effect. It is found that both geometrical configuration and bulk effect produce influence on the resonant frequency and admittance of the compound QCR system.

1. Introduction

Due to high sensitivity, simple structure, and easy interconnection with electronic measurement systems, thickness-shear mode (TSM) quartz crystal resonators (QCRs) have been widely used to monitor thin film deposition, and to characterize the mechanical behaviors of materials bounded onto the surface by measuring frequency shift and electric admittance spectrum [1,2]. For example, QCRs were applied to measure mass changes in materials attached on the surface in vapor phase [3], and employed to measure the viscoelastic properties of polymeric films and viscosity and density of Newtonian liquid [4–7]. In biological sensor applications, cells are often distributed on QCR surfaces to detect the evolution of complex shear module and sizes [8,9]. We note that in the above applications, the resonator system is usually simplified as a multilayer structure [10] and analyzed through the one-dimensional transmission line model (TLM) [11]. By measuring the electric admittance spectra, complex shear modulus $G = G' + jG''$ as well as average thickness of the loaded layer are extracted using TLM together with some curve-fitting technique [12]. As well-known, TLM treats the sensor structure as a combination of a QCR main-body and one or more isotropic, homogeneous non-piezoelectric layers. In the same time, it assumes that the physical fields induced by the TSM vibrations of a QCR system vary only in the thickness direction without dependence on the two lateral directions [10]. Obviously, the geometrical configurations of surface materials/structures will produce no influence on QCR vibrations when the surface units are homogenized into one or multi-layer films. Thus, TLM model is acceptable under the condition that the characteristic scale l_c of surface attached materials/structures is much smaller than QCR thickness h_Q, or materials/structures are more uniformly distributed on QCR surface. However, with the rapid development in QCR miniaturization, l_c gradually becomes comparable to h_Q, i.e., the geometrical configurations of surface materials/structures gradually become evident. Thus, it is required to develop proper two-dimensional models to describe the effect of geometrical configurations of surface loadings on the dynamic behavior of compound QCR systems.

In this paper, a two-dimensional model on the coupling thickness-shear vibrations of a QCR loaded by an array spherical-cap (SC) viscoelastic material-units has been established. To illustrate the effectiveness of our model, we calculated the admittance spectra of the tendon stem cells (TSCs) and compared with the existing experiment data. It is found that the calculation results of two-dimensional model are consistent with the corresponding experimental results, which indicates that our model has good veracity and reliability in analyzing the mechanical properties of SC shaped loadings. Following, we studied the admittance spectra

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when a QCR surface is covered by an array of Epoxy Resin (SU-8) SC units for different geometrical configurations and dimensions. Effect of geometrical configurations and bulk effect of surface materials/structures on the admittance spectra is investigated in detail.

2. Formulation of the problem

Fig. 1(a) schematically shows a QCR covered with an array of SC material units (cells, proteins, or polymer, etc.). The bottom radius and height of the spherical cap is denoted as r and h, respectively. For the thickness-shear vibrations of a resonator shown in Fig. 1(a), the governing equations of QCR [2] are

$$\begin{aligned} &T_{21,2} = \rho_Q \ddot{u}_1, \quad D_{2,2} = 0, \quad E_2 = -\phi_{,2}, \\ &T_{21} = c_{66} u_{1,2} + \eta_Q \dot{u}_{1,2} - e_{26} E_2, \quad D_2 = e_{26} u_{1,2} + \varepsilon_{22} E_2, \end{aligned} \tag{1}$$

where T_{21} is the non-zero stress component and u_1 is the non-zero displacement component, corresponding to the TSM vibrations. In addition, D_2 is the electric displacement and E_2 is the electric field. ρ_Q and η_Q stands for the mass density and viscosity of quartz. c_{66}, e_{26}, ε_{22} denote the elastic, piezoelectric and dielectric constants, respectively. Since the crystal plate is under harmonic vibrations, the time factor, $\exp(j\omega t)$, will be dropped in the following for simplicity. j stands for the imaginary unit, $\omega = 2\pi f$ stands for the driving frequency. The motion equations are obtained from Eq. (1)

$$\begin{aligned} &c_Q u_{1,22} = -\omega^2 \rho_Q u_1, \\ &e_{26} u_{1,22} - \varepsilon_{22} \phi_{,22} = 0, \end{aligned} \tag{2}$$

where

$$c_Q = c_{66} + e_{26}^2/\varepsilon_{22} + j\omega\eta_Q. \tag{3}$$

Then, we get the general solutions for (2) as

$$\begin{aligned} &u_1 = B_1 \sin k_Q x_2 + B_2 \cos k_Q x_2, \\ &\phi = (e_{26}/\varepsilon_{22})(B_1 \sin k_Q x_2 + B_2 \cos k_Q x_2) + B_3 x_2 + B_4, \\ &T_{21} = c_Q (B_1 k_Q \cos k_Q x_2 - B_2 k_Q \sin k_Q x_2) + e_{26} B_3, \end{aligned} \tag{4}$$

where B_1, B_2, B_3, B_4 are constants to be determined, and

$$k_Q = \omega(\rho_Q/c_Q)^{1/2}. \tag{5}$$

Following turns to analysis on the vibration of each single SC unit on QCR surface. It is reasonable to assume the deformation of each unit primarily in x_1-direction when the QCR is under the TSM vibrations. Since the size of a SC is much smaller than QCR, it is reasonable to approximately regard the cross-section of a SC as a spot on the QCR surface for simplicity. We therefore take the infinitesimal element as shown in Fig. 1(b). The motion equation of the element can be deduced as follows

$$\begin{aligned} &\tau(y+dy,t)\pi\left[R^2 - (R-h+y+dy)^2\right] - \tau(y,t)\pi\left[R^2 - (R-h+y)^2\right] \\ &\quad = \rho_c \ddot{u}(y,t)\pi\left[R^2 - (R-h+y)^2\right]dy. \end{aligned} \tag{6}$$

Introducing two dimensionless parameters $y^* = y/h$, $u^* = u/h$ and applying the constitutive relation $\tau(y,t) = \mu_c u_{1,2} + \eta_c \dot{u}_{1,2}$ to (6) yields

$$\begin{aligned} &\left[y^{*2} + 2\left(\frac{R}{h} - 1\right)y^* - 2\frac{R}{h} + 1\right]\frac{d^2u^*}{dy^{*2}} + 2\left(y^* + \frac{R}{h} - 1\right)\frac{du^*}{dy^*} \\ &\quad + \eta\left[y^{*2} + 2\left(\frac{R}{h} - 1\right)y^* - 2\frac{R}{h} + 1\right]u^* = 0, \end{aligned} \tag{7}$$

where $\eta = h^2\rho_c\omega^2/G$, and $G = \mu_c + j\omega\eta_c$ stands for the shear modulus of the SC units. ρ_c, μ_c, η_c denote the density, elastic shear modulus and viscosity of the surface loadings, respectively.

The boundary and the interface conditions are

$$\begin{cases} (u_1, T_{12}A_0)|_{x_2=h_Q} = (hu^*, NA_0\tau\pi r^2)|_{y^*=0}, \\ \tau|_{y^*=1} = 0, \qquad T_{12}A_0|_{x_2=-h_Q} = 0, \\ \phi|_{x_2=h_Q} = -\phi_0, \qquad \phi|_{x_2=-h_Q} = \phi_0. \end{cases} \tag{8}$$

where $r = \sqrt{2Rh - h^2}$, R stands for the radius of the cap and A_0 for the electrode area of the quartz plate.

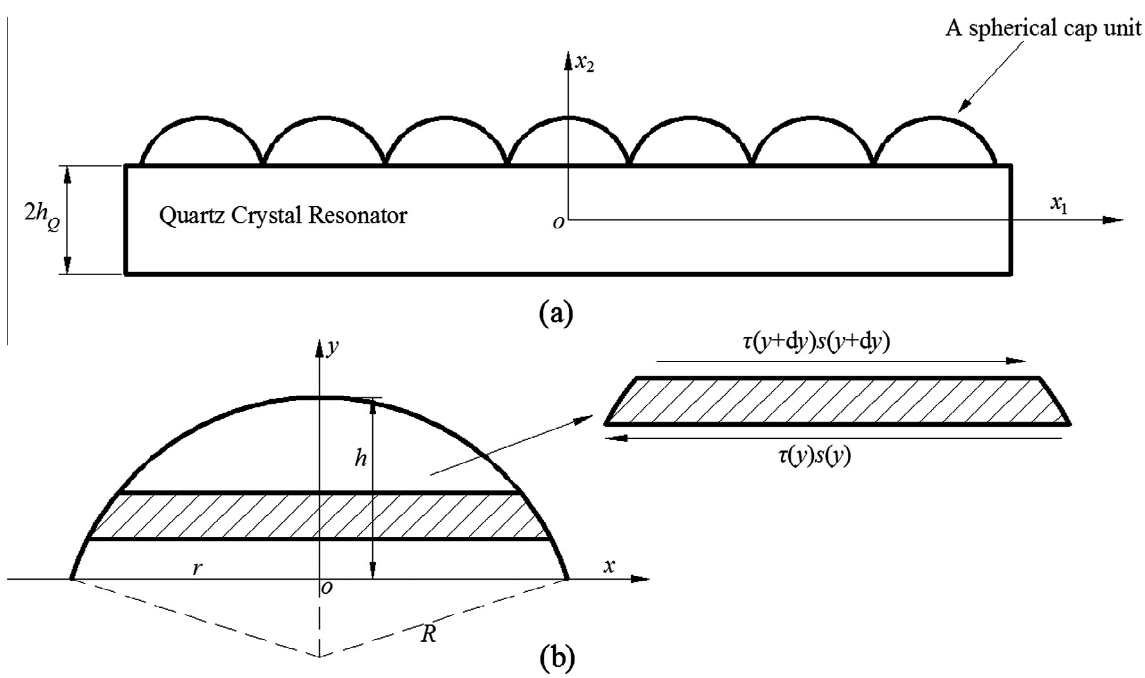

Fig. 1. (a) Schematic representation of a crystal plate covered by an array of spherical-cap material units. (b) The infinitesimal element on a spherical-cap material unit.