Ultrasonics 71 (2016) 1–11

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Acoustic radiation force and torque exerted on a small viscoelastic particle in an ideal fluid

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A R T I C L E I N F O

Article history:
Received 7 January 2016
Received in revised form 20 May 2016
Accepted 20 May 2016
Available online 25 May 2016

Keywords:
Acoustic radiation force
Acoustic radiation torque
Viscoelasticity

A B S T R A C T

We provide a detailed analysis on the acoustic radiation force and torque exerted on a homogeneous viscoelastic particle in the long-wave limit (i.e. the particle radius is much smaller than the incident wavelength) by an arbitrary wave. We assume that the particle behaves as a linear viscoelastic solid, which obeys the fractional Kelvin–Voigt model. Simple analytical expressions for the radiation force and torque are obtained. The developed theory is used to describe the interaction of acoustic waves (traveling and standing plane waves, and zero- and first-order Bessel beams) in the MHz-range with polymeric particles, namely lexan, low-density (LDPE) and high-density (HDPE) polyethylene. We found that particle absorption is chiefly the cause of the radiation force due to a traveling plane wave and zero-order Bessel beam when the frequency is smaller than 5 MHz (HDPE), 3.9 MHz (LDPE), and 0.9 MHz (lexan). Whereas in a standing wave field, the radiation force is mildly changed due to dispersion inside the particle. We also show that the radiation torque caused by a first-order Bessel beam varies nearly quadratic with frequency. These findings may enable new possibilities of particle handling in acoustophoretic techniques.

1. Introduction

Noncontact manipulation of particles including cells, and other microorganisms by means of the acoustic radiation force has become a promising method in biotechnology [1–10] and acoustic levitation in air [11–14]. This method is label-free and depends only on the mechanical properties of the particle and host fluid. Moreover, apart from being translated or trapped, a particle can be set to spin as a result of the acoustic radiation torque [15–17]. Consequently, a rotational degree of freedom is also available in particle manipulation based on acoustic methods.

Time-harmonic waves exerts a time-averaged (over the wave period) force known as the acoustic radiation force (ARF) on a suspended object through linear momentum transfer [18,19]. Seminal works have established the theoretical grounds to analyze the ARF applied on a spherical particle in a nonviscous fluid, and caused by waves having simple character, such as plane and spherical waves [20–29]. In recent years, extensive theoretical analysis has been accomplished on the ARF caused by spherically focused beams [30–33], Gaussian beams [34,35], non-diffracting and vortex beams [36–54], and a linear array [55]. Additionally, the ARF between two or more non-absorbing particles has also been analyzed [56–62].

Acoustic wave interaction with a suspended object in an inviscid fluid may also generate a time-averaged torque, known as the acoustic radiation torque (ART), by angular momentum transfer. Theoretical investigations of the ART can be found in Refs. [63–66]. Unlike ARF, the induced ART can only happen on absorbing objects.

n acoustophoretic applications, which include acoustical tweezers, levitators, and acoustofluidics devices, it is common that the handled particle has radius a much smaller than the acoustic wavelength λ. This corresponds to the so-called Rayleigh scattering regime (i.e. the long-wave limit). This limit is readily found in acoustofluidics devices operating at 2 MHz and handling microparticles as small as 1 μm in a water-like medium [7]. Moreover, particles like biological cells or polymers behaves as linear viscoelastic solid under a low-amplitude applied stress [67]. Therefore, a broader investigation on how the viscoelasticity of the particle affects the ARF and ART in the Rayleigh approximation is desired. This gave us the motivation to theoretically analyze these phenomena actuating on small viscoelastic particles in consideration of traveling and standing plane waves and zero- and first-order Bessel beams.

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http://dx.doi.org/10.1016/j.ultras.2016.05.018
0041-624X/

This work is not the first to present an investigation on the ARF and ART exerted on a viscoelastic particle. The ARF exerted on a homogeneous polyethylene sphere was also investigated [68]. Moreover, the ARF [69–71] and ART [72] acting on a viscoelastic spherical shell were also studied. In these articles, the analysis was limited to particles positioned in the beam axis. The ARF and ART were computed through a truncated series of the scattering coefficients, i.e. the coefficients in the partial-wave expansion of the scattered wave. Certainly, the Rayleigh scattering limit is present in such analysis. However, it is not possible to directly draw a relation between the particle's mechanical properties and the ARF and ART. Such connection is more than desirable: it gives a clearer picture on how acoustophoretic devices may handle suspended particles. We thus extend here the investigations performed in Refs. [68,72] to include a beam with arbitrary wavefront. Furthermore, we derive analytical expressions of ARF and ART, in the Rayleigh limit, based on the mechanical properties of viscoelastic particles. In particular, the fractional Kelvin–Voigt model is used to describe the particle viscoelasticity [73]. We also assume that thermoviscous effects are negligible: the host fluid is regarded as inviscid. This assumption requires that the particle radius a should be much larger than the thermal $\delta_t = \sqrt{2D_t/\omega}$ and viscous $\delta_v = \sqrt{2\nu_0/\omega}$ boundary layers [74], where D_t is the thermal diffusivity and ν_0 is the kinematic viscosity. In other words, $\delta_t, \delta_v \ll a \ll \lambda$. Typical values of the boundary layers in water at 2 MHz are $\delta_t = 0.15$ µm and $\delta_v = 0.38$ µm.

This paper is organized as follows. In Section 2, we present the theory of acoustic wave scattering by a spherical particle. In turn, scattering is directly connected to the ARF and ART phenomena [75]. In Section 3, we derive the longitudinal and shear wave equations stemming from the stress–strain relation based on the fractional Kelvin–Voigt model [73,76], i.e. a generalized Hooke's law with lossy fractional time-derivative terms. This model was chosen because it can describe the frequency power-law dependence experimentally observed on several viscoelastic materials [77–79]. In Section 4, we derive the scaled scattering coefficients to the monopole and dipole approximation that will be used in the partial-wave expansion formulas for the ARF and ART. In Section 6, we describe how to obtain the ARF and ART formulas to the monopole–dipole approximation. In Section 7, we present simple analytic expressions for the ARF and ART exerted on a polymeric particle in water at room temperature by traveling and standing plane waves, and zero- and first-order Bessel beams. In this analysis, three polymers are considered, namely lexan (commercial name for polycarbonate plastic), low- (LDPE) and high-density polyethylene. We established the frequency range to which absorption effects is the main cause of ARF due to traveling plane wave and Bessel beams for these particles. Additionally, we computed the ART on the polymeric particles due to a first-order Bessel beam. Finally, we summarize and conclude our analysis in Section 8.

2. Scattering theory

Consider a time-harmonic acoustic wave with angular frequency ω propagating in a homogeneous fluid of density ρ_0 and adiabatic speed of sound c_0. A spherical particle of radius a made of a viscoelastic material is suspended in the wavepath – see Fig. 1. Hence, the incident wave is subsequently scattered by the particle.

The incident and scattered waves are described in terms, respectively, of the velocity potentials $\phi_{\rm in}(\boldsymbol{r})e^{-i\omega t}$ and $\phi_{\rm sc}(\boldsymbol{r})e^{-i\omega t}$, as functions of position vector $\boldsymbol{r}$ and time t. The velocity potential amplitude functions satisfy the Helmholtz wave equation

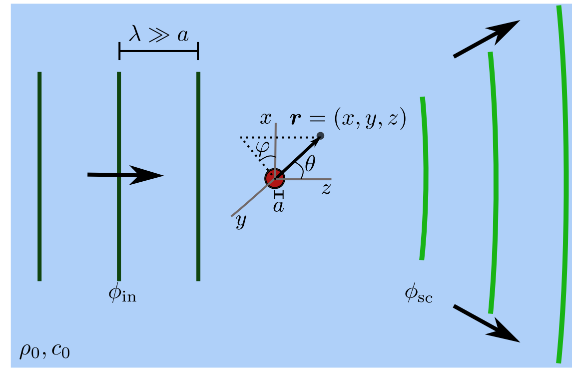

Fig. 1. An incident beam (dark-green vertical bars), with wavelength λ and represented by the velocity potential amplitude $\phi_{\rm in}$, hits a small viscoelastic particle (red circle) of radius a suspended in a fluid with density ρ_0 and speed of sound c_0. A Cartesian coordinate system is set in the particle's center with an observation point denoted by $\boldsymbol{r} = (x, y, z)$. Spherical coordinates (r, θ, φ) are also illustrated. The scattered waves (light-green arcs) are represented by the velocity potential amplitude $\phi_{\rm sc}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$(\nabla^2 + k^2)\begin{pmatrix}\phi_{\rm in}\\ \phi_{\rm sc}\end{pmatrix} = 0, \tag{1}$$

where $k = \omega/c_0$ is the wavenumber. In the linear approximation, the incident (scattered) pressure and fluid velocity are related by

$$p_{\rm in(sc)} = i\rho_0\omega\phi_{\rm in(sc)}, \tag{2}$$

$$\boldsymbol{v}_{\rm in(sc)} = \nabla\phi_{\rm in(sc)}. \tag{3}$$

Assume that the origin of the coordinate system is set in the sphere's center. Due to the symmetry of the problem, we describe the incident and scattered potentials as functions of spherical coordinates: radial distance r to the observation point $\boldsymbol{r} = (x, y, z)$, polar angle θ, and azimuthal angle φ. The potential functions can be expanded in a partial-wave series as follows [80]

$$\phi_{\rm in}(kr, \theta, \varphi) = \sum_{n,m} a_{nm} j_n(kr) Y_n^m(\theta, \varphi), \tag{4}$$

$$\phi_{\rm sc}(kr, \theta, \varphi) = \sum_{n,m} s_n a_{nm} h_n^{(1)}(kr) Y_n^m(\theta, \varphi), \tag{5}$$

where $\sum_{n,m} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}$, a_{nm} is the beam-shape coefficient of the incident wave [81], s_n is the scaled scattering coefficient, j_n is the nth-order spherical Bessel function, $h_n^{(1)}(kr)$ is the nth-order spherical Hankel function of first-type, and Y_n^m is the spherical harmonic of nth-order and mth-degree. Note that the scattering potential should satisfy the Sommerfeld radiation condition in order to ensue that no wave reflection occurs at infinity.

The beam-shape coefficients can be determined by inverting the partial-wave series in Eq. (4) through the orthogonality relation of the spherical harmonics. On the other hand, the scaled scattering coefficients will be determined by applying appropriate boundary conditions on the fluid-viscoelastic interface at the particle's surface.

3. Fractional Kelvin–Voigt model

The scattering particle is assumed to behave as a solid viscoelastic material. It is well established that the acoustic absorption of longitudinal (ℓ) or shear waves (s) in a wide range of viscoelastic materials obeys a power-law of frequency variation over several decades [82],

$$\alpha_j(\omega) = \alpha_{0,j}\omega^{y_j}, \quad j \in \{\ell, s\}, \tag{6}$$