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Level repulsion of GHz phononic surface waves in quartz substrate with finite-depth holes

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ABSTRACT

This paper presents numerical and experimental results on the level repulsion of gigahertz surface acoustic waves in an air/ST-cut quartz phononic structure with finite-depth holes. The colorful dispersion with the parameter of the in-plane (sagittal plane) ratio of polarization was adopted to determine the Rayleigh wave bandgap induced by the level repulsion. The results of numerical analyses showed that the frequency and width of the bandgap induced by the level repulsion strongly depend on the geometry of the air holes in the phononic structure. In the experiment, a pair of slanted interdigital transducers with frequency in the gigahertz range was designed and fabricated to generate and receive broadband Rayleigh waves, whereas the reactive ion etching process with electron-beam lithography was used to fabricate submicrometer phononic structures. The measured results of the bandgap induced by the level repulsion agreed favorably with the numerical prediction.

1. Introduction

The propagation of surface acoustic wave (SAW), bulk acoustic wave (BAW) and Lamb waves in phononic structures has been a topic of interest for more than a decade. In the literature, band structures of SAWs in semi-infinite two-dimensional (2D) phononic crystals (PCs) consisting of cubic [1] and triclinic [2] materials and, subsequently, cases of SAW modes in semi-infinite 2D piezoelectric PCs [3–5], have been studied using the plane wave expansion method. SAW bandgaps in PCs were first experimentally evidenced by Wu et al. [6] and Benchabane et al. [7] with frequency in the range of hundreds of megahertz. Sun and Wu [8] combined a finite-difference time-domain method with a perfect matching layer to study the band structures of SAW and BAW simultaneously. However, etching infinite-depth holey structures is impossible on a semi-infinite substrate. Numerous studies have investigated SAWs in finite-depth holey PCs by using the finite element method [9–11]. For applying PCs, a design that combines two-port SAW devices and PCs acting as reflective gratings was also demonstrated [12]. For cases of propagation of Lamb waves in phononic structures, readers are referred to a review article on phononic plate waves by Wu et al. [13].

Level repulsion, which avoids the crossing of eigenvalues in a wave propagation system, has also been a topic of interest in the PC community in recent years. Wu and Huang [14] reported the level repulsions of BAWs in 2D PCs and demonstrated that the polarizations of different wave modes could be used as the criterion for distinguishing real or apparent cross points in band structures. Achaoui et al. [15] investigated the polarization of BAWs in 2D piezoelectric PCs and phononic waveguides, and found that in addition to the strong coupling induced by the polarization states, there was a weaker but non-negligible coupling that originated from the material anisotropy. In a study of stubbed sonic crystal waveguides [16], the level repulsion between the symmetric and antisymmetric bands was identified and appeared as an evanescent mode connecting both bands. The aforementioned studies on level repulsions are all related to BAWs, whereas the level repulsion of SAWs in phononic structures warrants further study.

This paper presents numerical and experimental studies on the level repulsion of gigahertz SAWs in an air/ST-cut quartz phononic structure with finite-depth holes. The PC was formed by etching holes in square lattice in an ST-cut quartz wafer. In addition to the well-known bandgap induced by Bragg scattering, under certain specific geometrical conditions, a bandgap is formed by the level repulsion of the fundamental modes of Rayleigh and surface transverse waves.

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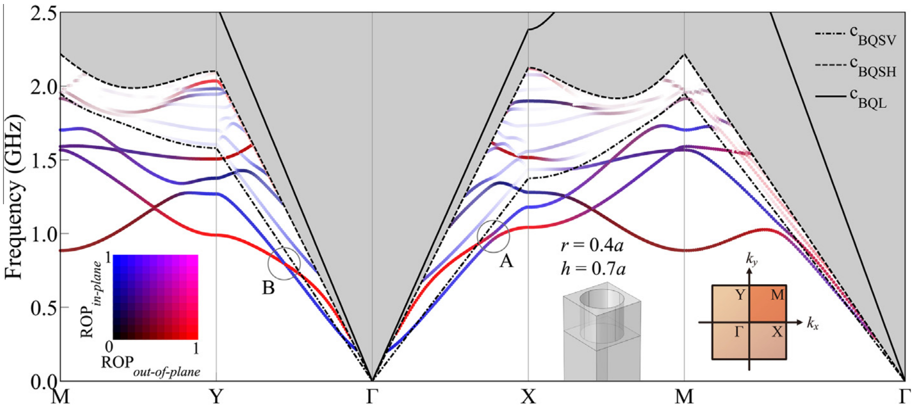

Fig. 1. Band structure of the PC with $r = 0.4a$, $h = 0.7a$. The red ones are closer to the surface transverse wave modes, whereas the blue ones are closer to the Rayleigh wave modes. The gray region represents the sound cone of ST-cut quartz. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. Analysis of level repulsions in an ST-cut quartz substrate with finite-depth holes

In this study, the finite element method (COMSOL Multiphysics) [17] was adopted in the numerical simulations of the band structures. Fig. 1 shows the calculated band structure of a square-latticed air/ST-cut quartz phononic structure, where the solid, dashed, and dashed dotted lines are the quasi-longitudinal, quasi-shear-horizontal, and quasi-shear-vertical BAWs, respectively. The lattice constant of the square lattice (a) is 1.2 μm, and the radius (r) and depth (h) of the holes are $0.4a$ and $0.7a$, respectively. To simulate the semi-infinite domain, the depth of the quartz substrate was set at 10-fold the hole depth. In the figure, the polarizations of SAW modes are indicated by the colors and opacity by using the method reported previously [18]. Near the Γ point, the wavelength approaches infinity; therefore, instead of SAW modes, the calculated modes are quasi-plate modes. Although the quasi-plate modes appeared because of the finite-depth substrate of the simulation model, it caused no misjudgment related to SAW modes insofar as the frequency considered was not located in the extremely low frequency region. To precisely describe SAW modes in the band structure, the out-of-plane (sagittal plane) and in-plane ratios of polarizations, $\text{ROP}_{out\text{-}of\text{-}plane}$ and $\text{ROP}_{in\text{-}plane}$, were introduced and assigned as red and blue, respectively. In Fig. 1, the modes denoted in red have a larger component transverse to the sagittal plane, indicating that they are closer to the surface transverse modes, whereas the modes denoted in blue are closer to the Rayleigh modes; that is, a larger component is on the sagittal plane. For Rayleigh waves that propagate along the ΓX direction, the Bragg bandgap is located in the range of 1.18–1.28 GHz. By distinguishing between red and blue, a level repulsion appears in region A (Fig. 1) along the ΓX section; that is, the polarizations of the lowest two modes are exchanged. By contrast, for wave modes that propagate along the ΓY direction, the lowest Rayleigh and surface transverse modes are decoupled and hence there is no level repulsion in region B.

Regarding only the lowest two modes, the partial figure in region A is shown in Fig. 2. The polarizations of the lowest two modes are exchanged without crossing. At the exchange point, the lowest two modes blend with the Rayleigh mode and the surface transverse mode. This represents that a level repulsion appeared. To determine whether there is a Rayleigh wave bandgap induced by the level repulsion, $\text{ROP}_{in\text{-}plane}$ should be quantized. When $\text{ROP}_{in\text{-}plane}$ at the exchange point is low enough (i.e., lower than a critical value), only a few of the Rayleigh wave can propagate through the phononic structure; that is, most Rayleigh wave can be prevented from propagating through the phononic structure. Hence, the level repulsion induces a Rayleigh wave bandgap. The critical value was determined according to the observation of the numerical transmission experiments using different critical values. Results showed that the bandgap width induced by the level repulsion agreed well with that from the numerical transmission experiment when the critical value is 0.5.

To determine the center frequency and bandgap width of the bandgap induced by the level repulsion, we calculated the $\text{ROP}_{in\text{-}plane}$ of the lowest two modes as a function of the frequency, as shown in Fig. 3. The cross of the two modes provides the center frequency of the bandgap. The width at which the $\text{ROP}_{in\text{-}plane}$ is 0.5 provides the bandgap width. In this case, the Rayleigh wave bandgap induced by the level repulsion is located at 0.95–1.00 GHz (as shown in pink[1] in Fig. 3), and the center frequency is located at 0.98 GHz.

To study the influences of the filling fraction and depth of the holes on the formation of level repulsion, a series of numerical calculations were performed. Fig. 4 shows the calculated band structures of SAWs along the ΓX direction with different hole radii and depths, which are (a) $r = 0.3a$, $h = 0.7a$; (b) $r = 0.4a$, $h = 0.7a$; (c) $r = 0.3a$, $h = 1.0a$; and (d) $r = 0.4a$, $h = 1.0a$. As shown in Fig. 4(a) and (c), when the radius of the holes is short ($0.3a$), there is no interaction between the lowest surface transverse (red) and Rayleigh (blue) modes and hence no level repulsion. When the hole radius is increased to $0.4a$, as shown in Fig. 4(b) and (d), the surface transverse mode approaches the Rayleigh mode and allows the level repulsion to occur. In addition, the results also showed that an increase in the hole depth reduces the center frequency of the level repulsion.

Fig. 5(a) and (b) show the center frequency and bandgap width of the bandgap induced by the level repulsion as functions of the radius and depth of the holes, respectively. As shown in Fig. 5(a), for the case of $h = 1.0a$, both the center frequency and the bandgap width decrease as the hole radius increases. For a hole radius shorter than $0.34a$, there is no level repulsion; the minimum filling fraction ensures the occurrence of the level repulsion. Setting $r = 0.4a$, Fig. 5(b) shows that as the hole depth increases, the center frequency of the level repulsion decreases and the bandgap width increases. The results showed that no level repulsion occurred for $h \leqslant 0.55a$, and the bandgap width saturated at approximately 5.3% of the center frequency. Additional numerical results showed that with deeper holes, the band structures became more complex,

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[1] For interpretation of color in Figs. 3 and 4, the reader is referred to the web version of this article.