Ultrasonics 71 (2016) 199–204

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Methodology for determining material constants of anisotropic materials belonging to the transversely isotropic system by ultrasound method

Wojciech Piekarczyk *, Dariusz Kata

AGH University of Science and Technology, Faculty of Materials Science and Ceramics, al. Mickiewicza 30, 30-059 Krakow, Poland

ARTICLE INFO

Article history:
Received 30 December 2015
Received in revised form 13 June 2016
Accepted 27 June 2016
Available online 29 June 2016

Keywords:
Elastic constants
Through-transmission method
Anisotropy
Polycrystalline graphite
Alumina – hexagonal boron nitride

ABSTRACT

The paper presents the methodology and results of the ultrasound determination of material constants of anisotropic materials belonging to the transversely isotropic system.

Ultrasound through-transmission method was used for determining material constants. Based on the measurements of velocities of longitudinal and transverse ultrasounds waves propagation, respectively polarized in required directions all the elastic and the material constant of the test materials were determined. Measurements of all the velocities necessary to determine the elastic constants were performed on a specially prepared individual samples.

The tests were carried out on porous polycrystalline anisotropic graphites of anisotropy in Young's modulus of up to 26% and Al_2O_3 composites with up to 30% of hBN causing anisotropy of Young's modulus of up to 50%.

It was found that for all tested samples the value of Young's modules and modules stiffness decreasing with increasing porosity in the graphites and increasing content of hBN in Al_2O_3.

1. Introduction

The elastic constants expressed by material constants (i.e. Young's modulus, shear modulus and Poisson's ratio) are ones of the most important parameters determining the mechanical properties of a material. They can be determined with static and dynamic methods [1]. Determining material constants with a static method is sufficient for isotropic materials. However, in the case of anisotropic materials, calculating all the material constants is complicated and sometimes impossible.

The role played by the elastic constants in the characteristics of mechanical properties of material enabled using ultrasonic methods not only for composite materials [2–6], single crystals or ceramic materials [7–10], but also for other materials having anisotropy, such as carbon fibers [11,12] and natural materials: bones [13,14], rocks [15,16] and wood [17,18].

2. Preparation and testing methods

2.1. Methodology

To characterize the elastic properties of anisotropic material with transversely isotropic symmetry there should be given 5 independent elastic constants C_{11}, C_{12}, C_{13}, C_{33}, C_{44} or 7 material constants E_{33}, E_{11}, G_{31}, G_{12}, μ_{31}, μ_{23}, μ_{12}, of which two are dependent [19]. The elastic constant matrix is the same for hexagonal symmetry. All the elastic constants and material constants can be determined by measurements the velocity of longitudinal and transverse waves that propagate in the appropriate directions of the sample. Additionally measurements of transverse wave must be provided for appropriate polarization related to the direction of sample orientation [5,6,20].

The elastic constant C_{ij} of anisotropic materials with hexagonal symmetry have been related to the density, the direction of the axis of symmetry and the velocities of ultrasonic waves propagation, by the relationships known in the literature since the middle of twentieth century [21–23].

For waves propagating at an angle θ to the axis of symmetry of the hexagonal system there can occur three different types of ultrasonic waves which are described by the following equations:

* Corresponding author.
E-mail address: wpiekar@agh.edu.pl (W. Piekarczyk).

http://dx.doi.org/10.1016/j.ultras.2016.06.015
0041-624X/

$$\rho V_L(\theta)^2 = \frac{1}{2}[(C_{11}+C_{44})\sin^2\theta + (C_{33}+C_{44})\cos^2\theta + \varphi(C_{ij},\theta)], \quad (1)$$

$$\rho V_{\parallel}(\theta)^2 = \frac{1}{2}\left[(C_{11}+C_{44})\sin^2\theta + (C_{33}+C_{44})\cos^2\theta - \varphi(C_{ij},\theta)\right], \quad (2)$$

$$\rho V_{\perp}(\theta)^2 = \frac{1}{2}(C_{11}-C_{12})\sin^2\theta + C_{44}\cos^2\theta \quad (3)$$

where V_L is the velocity of longitudinal wave.

$V_{\parallel}$ and $V_{\perp}$ are the velocities of transverse waves polarized parallel and perpendicular to the plane defined by the axis Z and the direction of propagation of the wave, θ is the angle between the symmetry axis and the direction of wave propagation.The function $\varphi(C_{ij},\theta)$ is given by:

$$\varphi(C_{ij},\theta) = \Big\{(C_{11}-C_{44})^2\sin^4\theta + (C_{33}-C_{44})^2\cos^4\theta + 2\sin^2\theta\cos^2\theta\left[(C_{11}-C_{44})(C_{44}-C_{33}) + 2(C_{13}+C_{44})^2\right]\Big\}^{1/2} \quad (4)$$

These dependencies for the homogenous material (constant density) are correct, when the propagating ultrasonic waves are significantly (several times) larger than the dimensions of inhomogeneous in the sample (grains, pores, fibers), or if the group velocity is equal to the phase velocity [24,25]. At the same time the test sample should fulfill the conditions of the three-dimensional medium, i.e. all the dimensions of the sample are similar or for an extended sample – the smallest of its dimension is several times greater than the length of the ultrasonic wave used [26].

The most commonly used measurement is to measure the velocity of the waves at the angle $\theta = 45°$ to the axis of symmetry, for which general equations are considerably simplified. It happens that, in order to check the dispersion in the material or as comparative tests measurements are also made at other angles [2,7,27].

With the above assumptions, relationships allow to determine elastic constants of transverse isotropic material with measurements of the velocities of particular ultrasonic waves propagation along the main axis of the system and at an angle $\theta = 45°$, are as follows [19]:

$$C_{11} = \rho V_{1/1}^2 \quad (5)$$

$$C_{33} = \rho V_{3/3}^2 \quad (6)$$

$$C_{44} = \rho V_{1/3}^2 \quad (7)$$

$$C_{66} = \rho V_{1/2}^2 = \rho V_{2/1}^2 \quad (8)$$

$$C_{12} = C_{11} - 2C_{66} = \rho\left[V_{1/1}^2 + 2V_{1/3}^2 - 4V_{13/2}^2\right] \quad (9)$$

$$C_{13} = \left[\left(C_{11}+C_{44}-2\rho V_{13/13}^2\right)\left(C_{44}+C_{33}-2\rho V_{13/13}^2\right)\right]^{1/2} - C_{44} \quad (10)$$

$V_{1/1}$ – velocity of a longitudinal wave propagating in the direction of axis 1 with particle motion in the direction of axis 1.
$V_{1/2}$ – velocity of transverse wave propagating in the direction of axis 1 with particle motion in the direction of axis 2.
$V_{13/2}$ – velocity of transverse wave propagating in the plane of 1–3 (at an angle of 45°) with particle motion in the direction of axis 2.
$V_{13/13}$ – velocity of longitudinal or transverse (quasi-longitudinal and quasi-transverse) waves propagating in the plane of 1–3 (at an angle of 45°) with particle motion also in the plane 1–3.

The C_{12} elastic constant can be determined directly from the measurements along the major axis and the aforementioned equation (Eq. (9)): $C_{66} = (C_{11} - C_{12})/2$. However, with the prepared sample (ground down corner edges) elastic constant C_{12} can also be determined based on equation taking into account the velocity of the transverse wave $V_{13/2}$ (Eq. (9)), which provides a possibility to verify the obtained values.

Preparation of a single sample for ultrasound tests of anisotropic material belonging to the transversely isotropic class, so that you can perform on it all possible measurements of the velocities of propagation of all possible ultrasonic wave is to cut down or truncating opposite edges at an angle of 45° to the axis of symmetry.

Below in Fig. 1 is a schematic preparation of the sample cube.

In the case of truncating two mutually perpendicular edges there is a possibility to obtain 15 independent measurements of velocity waves from a single sample of which is equivalent to – allows to verify the values obtained.

There are equivalent velocities for transversely isotropic material:

$V_{1/1} = V_{2/2}$;
$V_{1/2} = V_{2/1}$;
$V_{1/3} = V_{3/1} = V_{2/3} = V_{3/2}$;
$V_{13/13} = V_{23/23}$;
$V_{13/2} = V_{23/1}$

The material constants for the transversely isotropic class are expressed by the elastic constants in respective relationships [27].

$$E_{\parallel(33)} = S_{33}^{-1} = \left[C_{33}(C_{11}+C_{12}) - 2C_{13}^2\right] \Big/ (C_{11}+C_{12}) \quad (11)$$

$$E_{\perp(11)} = S_{11}^{-1} = \left[(C_{11}-C_{12})\left(C_{11}C_{33}+C_{12}C_{33}-2C_{13}^2\right)\right] \Big/ \left(C_{11}C_{33}+2C_{13}^2\right) \quad (12)$$

$$G_{\parallel(31)} = S_{44}^{-1} = C_{44} \quad (13)$$

$$G_{\perp(12)} = S_{66}^{-1} = \frac{1}{2}(S_{11}-S_{12})^{-1} = C_{66} = \frac{1}{2}(C_{11}-C_{12}) \quad (14)$$

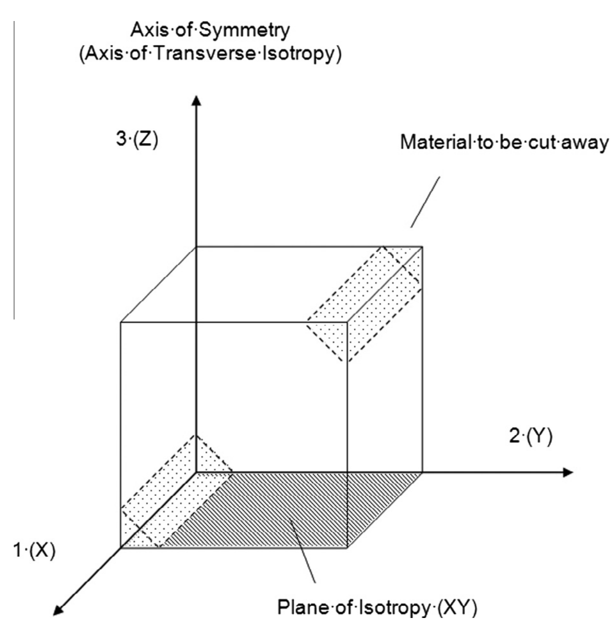

Fig. 1. Sample's scheme preparation of anisotropic material belonging to the transversely isotropic class for ultrasound tests.