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A numerical study of non-collinear wave mixing and generated resonant components

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ABSTRACT

Interaction of two non-collinear nonlinear ultrasonic waves in an elastic half-space with quadratic nonlinearity is investigated in this paper. A hyperbolic system of conservation laws is applied here and a semi-discrete central scheme is used to solve the numerical problem. The numerical results validate that the model can be used as an effective method to generate and evaluate a resonant wave when two primary waves mix together under certain resonant conditions. Features of the resonant wave are analyzed both in the time and frequency domains, and variation trends of the resonant waves together with second harmonics along the propagation path are analyzed. Applied with the pulse-inversion technique, components of resonant waves and second harmonics can be independently extracted and observed without distinguishing times of flight. The results show that under the circumstance of non-collinear wave mixing, both sum and difference resonant components can be clearly obtained especially in the tangential direction of their propagation. For several rays of observation points around the interaction zone, the further it is away from the excitation sources, generally the earlier the maximum of amplitude arises. From the parametric analysis of the phased array, it is found that both the length of array and the density of element have impact on the maximum of amplitude of the resonant waves. The spatial distribution of resonant waves will provide necessary information for the related experiments.

1. Introduction

Evaluation of structural damage and monitoring of structural health condition for mechanical, civil and aerospace systems have attracted much attention in the last few decades, for they are clearly indicative of lifetime estimation and operative costs of industrial equipment and machinery. Ultrasonic wave technique is one of the most effective methods of modern nondestructive evaluation (NDE) and capable of revealing material properties through wave propagation. Compared with conventional linear ultrasonic methods, nonlinear methods are more sensitive to the progressive degradation and microstructure damages at early stage of materials. Several nonlinear acoustic phenomena have been applied to evaluate structural damage, including higher harmonic generation, sub-harmonic generation, shift of resonance frequency and mixed frequency response [1].

Fatigue is one of the most common material degradation mechanisms in industry. Dislocations due to fatigue progression cause a nonlinear distortion into a propagating ultrasonic wave in solids and generate higher harmonics which manifests themselves in nonlinearity parameters. Second harmonic generation [2] is a frequently adopted method to measure second order acoustic nonlinearity parameter β. The average β over a wave propagating distance is measured with this method after which system nonlinearity and sample material nonlinearity have to be separated away carefully.

Wave mixing is another effective way to observe the nonlinearity parameter of the material. Interaction caused by material nonlinearities between two intersecting ultrasonic waves was firstly discovered in 1960s [3–5]. A third wave with a frequency and wave vector equal to the sum or difference of the incident wave frequencies and wave vectors can be generated when meeting certain conditions. Recently the interaction cases have also been investigated and summarized theoretically by Kuvshinov et al. [6] and Korneev et al. [7], and in the linear dependence between the amplitude of the resonant wave and the value of the nonlinear material parameters has been established. The method has several advantages compared with other nonlinear techniques. Firstly, system nonlinearity is excluded because only the material nonlinearity in the interacting zone gets involved in the measured signal. Secondly, it is convenient to observe the nonlinearity in a certain region by designing a corresponding intersection. Thirdly, the resonant component of the signal can be separated from harmonics of incident waves by their frequency difference.

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Cases of two collinear mixing waves were implemented numerically and experimentally by Liu et al. [8,9] to measure the nonlinearity parameter. Under this circumstance, two sensors have to be placed at a pair of parallel boundary surfaces. However, there exist lots of situations in industry that sensors can be only put in a single side, thus the necessity of focusing on non-collinear interaction is naturally taken into consideration. The non-collinear mixing technique, developed by Croxford et al. [10] and categorized into the method of mixed frequency response mentioned above, has been proved experimentally to be able to measure material nonlinearity to assess plasticity and fatigue damage while system nonlinearities can be both independently measured and largely eliminated. Demoenko et al. [11] investigated the responses of the nonlinear ultrasonic wave mixing technique numerically that radiation patterns of the scattered mixed beam both at and away from the perfect resonant condition are computed, and verified that a kissing bond and subsurface micro-cracks in polymer plates are detected experimentally.

As for a non-collinear ultrasonic wave intersection, although the resonant condition and factors affecting the intensity of the scattered wave have been mentioned and discussed, there are still some basic questions unknown in this field. For instance, high primary frequencies, large primary wave amplitudes and a large interaction volume are believed to be essential to generate an observable scattered wave when the resonant condition is satisfied [3], but it is only a qualitative but not a quantitative description. Here the problem is considered that how large the interaction volume should be with two specific primary waves. As for a two-dimensional model, the volume could be characterized as an area which further depends on the widths of two beams set up on the boundary. Another quite important thing still unknown is what the spatial amplitude distribution of the scattered wave is. Under this circumstance, a numerical simulation focused on the wave mixing process is worth studying.

With a finite difference algorithm namely semi-discrete central scheme, this paper aims to investigate both the generation and propagation of a resonant wave, thus obtaining a better understanding of the nonlinear wave mixing phenomenon.

2. Model for wave propagation in an elastic half-space with quadratic nonlinearity

A pair of coupled hyperbolic partial differential equations describing the wave propagation in an elastic half-space with quadratic nonlinearity [12,13] is applied here. The finite difference model from [12,13] is adapted here to the current problem of the nonlinear interaction of two waves, and some equations are reminded here for completeness.

A Cartesian coordinate system utilized with an elastic half-space is built, and two time-dependent line loads are imposed on the boundary (see Fig. 1). The two-dimensional motion is governed by a second-order hyperbolic system of partial differential equations as follows:

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}=\rho\frac{\partial^2 u_x}{\partial t^2},\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}=\rho\frac{\partial^2 u_y}{\partial t^2} \tag{1}$$

$\sigma_{xx}=\sigma_{xx}(x,y,t)$, $\sigma_{yy}=\sigma_{yy}(x,y,t)$, $\sigma_{xy}=\sigma_{xy}(x,y,t)$ represents stress components, u_x and u_y represent displacement components in x and y directions, respectively, and ρ represents the density of the material.

The boundary conditions for the problem are given by

$$\begin{aligned}&\dot{u}_{x1}(0,y_1,t)=-QF_1(t)\delta(y),\quad \dot{u}_{x2}(0,y_2,t)=-QF_2(t)\delta(y),\\&\dot{u}_{y1}(0,y_1,t)=\dot{u}_{y2}(0,y_2,t)=0,\end{aligned} \tag{2}$$

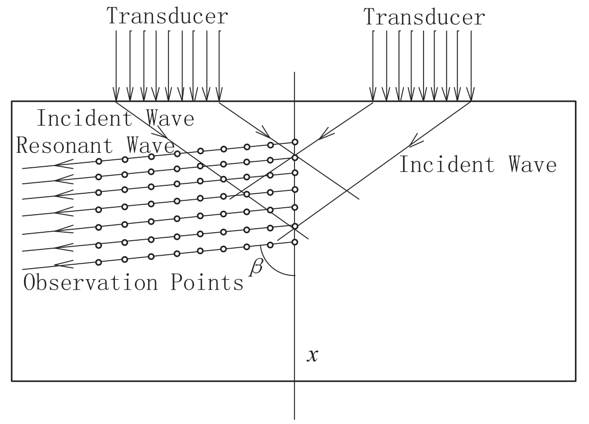

Fig. 1. Oblique incidence with observation points distributed in seven rays.

$\delta(y)$ is the delta function representing the line load, x_1, x_2, y_1, y_2 denote Cartesian coordinate components for two excitation sources respectively, $F_1(t)$ and $F_2(t)$ denote the input signal functions of two transducers respectively both of which can be given by

$$F(t)=\begin{cases}\frac{1}{2}\sin\left(2\pi\frac{t}{t_f}\right)\left(1-\cos\left(2\pi\frac{t}{t_f}\right)\right) & \text{if } t\leqslant t_f\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

Q is the amplitude of $F(t)$, and here it is chosen to be 10 m/s. A Hann window is applied to avoid energy leak. Displacements and velocities of particles in the half space are all set to be zero at time zero.

Here only small strain deformations are considered, and thus the corresponding constitutive equations can be written as

$$\begin{aligned}\sigma_{xx}&=(\lambda+2\mu)\frac{\partial u_x}{\partial x}+\lambda\frac{\partial u_y}{\partial y}+(l+2m)\left(\frac{\partial u_x}{\partial x}\right)^2+l\frac{\partial u_y}{\partial y}\left(2\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)+\frac{m}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)^2,\\ \sigma_{yy}&=(\lambda+2\mu)\frac{\partial u_y}{\partial y}+\lambda\frac{\partial u_x}{\partial x}+(l+2m)\left(\frac{\partial u_y}{\partial y}\right)^2+l\frac{\partial u_x}{\partial x}\left(2\frac{\partial u_y}{\partial y}+\frac{\partial u_x}{\partial x}\right)+\frac{m}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)^2,\\ \sigma_{xy}&=\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)\left(\mu+m\frac{\partial u_x}{\partial x}+m\frac{\partial u_y}{\partial y}\right),\end{aligned} \tag{4}$$

λ, μ are second-order elastic constants, and l, m are third-order elastic constants.

Phase steering is applied here by sequentially triggering the array elements. The direction of the acoustic beam propagation may be reoriented to any angle azimuthally only by altering the timing sequence of the excitation pulses [14]. In this research the phase steering technique is used to generate interacting ultrasonic waves in the simulation. There are 120 elements altogether in each array. Compared with the shortest wave length 0.6 mm considered in the problem, the size of each cell is 0.03 mm. The configuration of one phased array transducer is shown in Fig. 2 (Style 1).

3. Numerical solutions

The nonlinear problem discussed here can be represented in a hyperbolic system of conservation laws which is acted on a two-dimensional domain as follows,

$$\frac{\partial \mathbf{q}}{\partial t}+\frac{\partial \mathbf{f}(\mathbf{q})}{\partial x}+\frac{\partial \mathbf{g}(\mathbf{q})}{\partial y}=\mathbf{0} \tag{5}$$

$$\mathbf{q}(x,y,t)=[q^{(1)},q^{(2)},q^{(3)},q^{(4)},q^{(5)}]^T=[\dot{u}_x,\dot{u}_y,u_{x,x},u_{y,y},u_{x,y}+u_{y,x}]^T \tag{6}$$

$$\begin{aligned}\mathbf{f}(x,y,t)&=\left[-\frac{\sigma_{xx}(q^{(3)},q^{(4)},q^{(5)})}{\rho},-\frac{\sigma_{xy}(q^{(3)},q^{(4)},q^{(5)})}{\rho},-q^{(1)},0,-q^{(2)}\right]^T\\ \mathbf{g}(x,y,t)&=\left[-\frac{\sigma_{xy}(q^{(3)},q^{(4)},q^{(5)})}{\rho},-\frac{\sigma_{yy}(q^{(3)},q^{(4)},q^{(5)})}{\rho},0,-q^{(2)},-q^{(1)}\right]^T\end{aligned} \tag{7}$$