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On the existence of guided acoustic waves at rectangular anisotropic edges

Pavel D. Pupyrev [a,b], Alexey M. Lomonosov [a,b], Aleksandar Nikodijevic [b], Andreas P. Mayer [b,*]

[a] General Physics Institute, RAS, Moscow, Russian Federation
[b] HS Offenburg – University of Applied Sciences, 77723 Gengenbach, Germany

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ABSTRACT

The existence of acoustic waves with displacements localized at the tip of an isotropic elastic wedge was rigorously proven by Kamotskii, Zavorokhin and Nazarov. This proof, which is based on a variational approach, is extended to rectangular anisotropic wedges. For two high-symmetry configurations of rectangular edges in elastic media with tetragonal symmetry, a criterion is derived that allows identifying the boundary between the regions of existence for wedge modes of even and odd symmetry in regions of parameter space, where even- and odd-symmetry modes do not exist simultaneously. Furthermore, rectangular edges with non-equivalent surfaces are analyzed, and it is shown that at rectangular edges of cubic elastic media with one (110) surface and one (001) surface, a tip-localized guided wave always exists, apart from special cases that are characterized.

1. Introduction

Acoustic waves guided at the straight edge of an elastic medium were first discovered in numerical calculations by Lagasse [1] and Maradudin et al. [2] in the early seventies of the last century. Since then numerous experimental and theoretical investigations of such one-dimensionally guided waves have been carried out (for reviews see [3–6]), which have been termed acoustic wedge waves or line acoustic waves. Most of these studies were concerned with isotropic materials, and effects of anisotropy have rarely been addressed explicitly in the past ([7] and some works referred to in [5]).

The character of the displacement field of wedge waves (WWs) in anisotropic media may be very different from that in isotropic wedges. While in the latter, wedge acoustic waves may be characterized as even or odd/flexural, this is no longer possible in anisotropic wedges that lack reflection symmetry with respect to their mid-plane. Also, anisotropy leads to new phenomena like pseudo-wedge waves [8,9] and different types of wedge modes on two adjacent rectangular wedges at the same sample [9].

Wedge acoustic waves in ideal wedges made of a homogeneous material share with surface acoustic waves (SAW) in homogeneous media the property of being non-dispersive, i.e. their phase velocity does not depend on their frequency. Another interesting analogy is the problem of their existence in arbitrary propagation geometries of anisotropic media. In the case of surface acoustic waves, rigorous statements were established concerning their existence, involving criteria based on bulk wave properties [10,11]. In the case of wedge waves, a rigorous existence proof has so far been provided only for isotropic media [12,13]. In this work, we try to establish criteria for the existence of wedge waves in anisotropic rectangular wedges with certain symmetries. These criteria are based on surface acoustic wave properties.

Already in their pioneering study of WWs in a Poisson medium (i.e. an isotropic medium with equal Lamé constants), Moss et al. [14] found that these guided waves do not exist for all wedge angles. Recently, Kamotzki [12] has given a proof for the existence of WWs in isotropic wedges for wedge angles smaller than 90° and arbitrary Poisson ratio. This proof was extended by Zavorokhin and Nazarov [13], and these authors specified bounds for the existence domains of symmetric vs. anti-symmetric WWs in the two-dimensional parameter space with parameters Poisson ratio and wedge angle. These existence proofs are based on a variational principle, i.e. the Rayleigh quotient with suitable test functions. A motivation of our work was the question to what extent this approach may be extended to wedges made of anisotropic media and whether general statements can be made concerning the existence of WWs, based on SAW properties on the wedge faces.

Numerical studies for rectangular wedges reveal that, similar to surface acoustic waves, the character of the displacement field of wedge waves strongly varies with the orientation of the apex line

* Corresponding author.
E-mail address: andreas.mayer@hs-offenburg.de (A.P. Mayer).

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and the two surface normals in the same anisotropic elastic material. Examples are presented for the cubic crystals silicon, strontium fluoride and indium arsenide. While for certain orientations a wedge wave is found with displacement field strongly localized at the wedge tip, this is not the case for others, where the displacement field penetrates far into one of the two surfaces of the wedge, and it requires precise calculations to decide on the basis of numerical results whether a guided wave exists in these cases. The latter of the above-mentioned wedge configurations share the property of the two surfaces being not equivalent due to lack of mirror symmetry with respect to the wedge's midplane. They include the silicon wedge with apex direction [1 −1 0] and surface normal vectors along (1 1 0) and (0 0 1) and two non-equivalent configurations with apex direction [1 −1 0] and surface normal along (1 1 1) and (1 1 2).

The numerical calculations were carried out on the basis of an expansion of the displacement field in a double series of Laguerre functions [2,14,15]. This method is particularly suited for the investigations reported here as its accuracy and convergence properties are essentially controlled by only one parameter, namely the number of Laguerre functions included in the double series.

After presenting numerical results obtained with the Laguerre-function method mentioned above, the variational approach by Kamotzki, Zavorokhin and Nazarov for proving wedge wave existence in isotropic media is applied to acoustic waves propagating along the tip of rectangular anisotropic edges. With a straightforward generalisation of the test functions introduced by these authors, a criterion is derived for the existence of wedge waves in two rectangular wedge configurations with two equivalent surfaces and elastic media of tetragonal symmetry. This criterion is applied to the special case of cubic media and its predictions compared to the results of numerical calculations.

With a test function that is suitable for wedges with two non-equivalent surfaces, a sufficient condition is derived for the existence of wedge waves in anisotropic rectangular wedges. It is shown that this criterion is satisfied for two of the three aforementioned wedge configurations with non-equivalent surfaces in media of cubic symmetry.

2. Formulation of the problem

Propagation of acoustic waves is considered in an elastic wedge filling the spatial region $x_3 > 0,\ x_2 > x_3 \cot\theta$, where θ is the wedge angle ($0 < \theta < \pi$). The numerical examples for anisotropic wedges presented in Section 3 all refer the rectangular case ($\theta = \pi/2$). The apex direction of the wedge is along x_1. Acoustic wave propagation is described in the framework of elasticity theory involving the displacement field $(u_\alpha(x_1, x_2, x_3, t))$, which depends on the three spatial coordinates and on time t. The index $\alpha = 1, 2, 3$ labels the three Cartesian components. The displacement field has to satisfy the equation of motion

$$\rho \frac{\partial^2}{\partial t^2} u_\alpha = \frac{\partial}{\partial x_\beta} T_{\alpha\beta}. \tag{2.1}$$

Here and in the following, we invoke the convention of summation over repeated Cartesian indices. In (2.1), $(T_{\alpha\beta})$ is the stress tensor, which is expressed in terms of displacement gradients via

$$T_{\alpha\beta} = C_{\alpha\beta\mu\nu} \frac{\partial}{\partial x_\nu} u_\mu. \tag{2.2}$$

$(C_{\alpha\beta\mu\nu})$ is the tensor of the elastic moduli, referring to the coordinate system introduced above, and ρ is the mass density of the wedge material. In addition, the surfaces of the wedge have to be traction-free, i.e. for any point on the wedge faces, the quantities $T_{\alpha\beta}N_\beta,\ \beta = 1, 2, 3,$ have to vanish, where $N_\beta,\ \beta = 1, 2, 3,$ are the three components of a vector normal to the corresponding surface.The displacement field may be decomposed as

$$u_\alpha(x_1, x_2, x_3, t) = U_\alpha(kx_2, kx_3) \exp[ik(x_1 - \upsilon t)], \tag{2.3}$$

where k is a one-dimensional wave-vector and υ the phase speed along the apex line. The functions $U_\alpha(y, z)$ are solutions of the eigenvalue problem defined by the equations

$$\rho\upsilon^2 U_\alpha(y, z) = D_\beta \overline{T}_{\alpha\beta}(y, z), \tag{2.4}$$

where

$$\overline{T}_{\alpha\beta}(y, z) = C_{\alpha\beta\mu\nu} D_\nu U_\mu(y, z) \tag{2.5}$$

which have to be satisfied in the domain $z > 0,\ y > z\cot\theta$ along with the boundary conditions

$$\overline{T}_{\alpha 3}(y, 0) = 0, \tag{2.6}$$

$$\sin\theta \overline{T}_{\alpha 2}(z\cot\theta, z) - \cos\theta \overline{T}_{\alpha 3}(z\cot\theta, z) = 0. \tag{2.7}$$

The operator (D_α) is defined as

$$D_1 = i,\ D_2 = \partial/\partial y,\ D_3 = \partial/\partial z. \tag{2.8}$$

For the numerical analysis of rectangular wedges, we follow [2] and expand the functions $U_\alpha(y, z)$ in a series of products of Laguerre functions, $\varphi_n(x),\ n = 0, 1, 2, \ldots, N-1$,

$$U_\alpha(y, z) = \sum_{m,n=0}^{N-1} e^{(\alpha)}_{mn} \varphi_m(y) \varphi_n(z). \tag{2.9}$$

The velocities υ and expansion coefficients $e^{(\alpha)}_{mn}$ follow from the eigenvalues and eigenvectors of a Hermitian matrix,

$$\upsilon^2 e^{(\alpha)}_{mn} = \sum_{p,q=0}^{N-1} M^{(\alpha\beta)}_{mn\,pq} e^{(\beta)}_{pq}, \tag{2.10}$$

where

$$M^{(\alpha\beta)}_{mn\,pq} = \frac{1}{\rho} \int_0^\infty \int_0^\infty [D_\mu \varphi_m(y) \varphi_n(z)]^* C_{\alpha\mu\beta\nu} [D_\nu \varphi_p(y) \varphi_q(z)] dy dz. \tag{2.11}$$

(More details and an extension to non-rectangular wedges are described in [14–16].)

3. Numerical findings

Numerical calculations were carried out for rectangular wedges of anisotropic materials with cubic symmetry. The anisotropy can be characterized by the Zener ratio $A = 2c_{44}/(c_{11} - c_{12})$. (We follow here the convention that when carrying Voigt indices, the elastic moduli are denoted by lower-case c.)

In the case of silicon and coordinate axes along the crystallographic (cubic) axes, a well tip-localized wedge wave is found. An easily applicable criterion for the presence of a fully tip-localized wedge mode in the numerical calculations is the distance of the lowest eigenvalue in (2.4) and (2.5) from the continuum of surface and bulk modes. Alternatively, one may investigate the displacement field itself. To characterize the degree of edge-localisation, we define a penetration depth for each surface of a rectangular wedge in the following way:

$$d_2 = \frac{\int_0^\infty |\vec{U}(0, kx_3)|^2 x_3 dx_3}{\int_0^\infty |\vec{U}(0, kx_3)|^2 dx_3}, \tag{3.1}$$

$$d_3 = \frac{\int_0^\infty |\vec{U}(kx_2, 0)|^2 x_2 dx_2}{\int_0^\infty |\vec{U}(kx_2, 0)|^2 dx_2}. \tag{3.2}$$