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Effects of coupled interfacial imperfections on SH wave propagation in a layered multiferroic cylinder

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ABSTRACT

Although SH-wave propagation in layered multiferroic composites with imperfect interfaces has been widely investigated, the effects of interfacial imperfection couplings still remain as an un-tackled problem deserving studying. The present paper considered the SH-wave propagation in a bi-layered multiferroic cylinder that has inter-coupled magnetic, electric and mechanical imperfections on the interface. The dispersion equations of SH waves are derived and numerically solved. Parametric studies are conducted on the numerical results to reveal the effects of interfacial imperfections and their inter-couplings on phase velocity. Discussion here yields four main conclusions, which can serve as guidelines for the optimal design of multiferroic composites and the related smart devices.

1. Introduction

Ferromagnetic/ferroelectric (FM/FE) composites are the key materials of many non-destructive detection devices due to their prominent coupling of multi-physics [1]. For example, the kernel parts of most surface acoustic wave (SAW) sensors are frequently made of FM/FE ceramics [2–4]. Because the SAW sensors generally work under the action of SH waves, it is necessary to study the propagation behavior of SH waves in these devices, in order to improve their performances.

With the wider and wider applications of SAW sensors, the studies on SH-wave propagation under various working conditions have become hotspots in this field. SH waves propagating in FM/FE composites generally have dispersion behaviors, i.e., the phase velocity is dependent on the wave number [5]. Because the dispersion behavior is a significant factor affecting the performances of SAW sensors, most researchers in this area focus their attention on the dispersion relations. Qian et al. investigated a periodic piezoelectric (PE)-polymeric layered structure with SH waves propagating either along or normal to the interface, and revealed the filter effect based on the dispersion equations [6]. Calas et al. considered SH waves in a magnetoelectroelastic (MEE) heterostructure, and presented five limit cases of the dispersion relations [7]. Du et al. solved the problem of SH SAW propagating in a layered MEE cylinder, and calculated the phase velocity numerically under both the magneto-electrically open and shorted conditions [8]. Yuan et al. analyzed the effects of viscous liquid on the dispersion relation of SH SAW in a layered magneto-electric structures consisting of a piezomagnetic (PM) layer and a PE half plane [4]. Pang et al. discussed the dispersion behavior and band characteristics of SH waves in a periodically layered plate consisting of either the PE/PM/PE unit cell or the PM/PE/PM one [9]. Gaur and Rana surveyed the propagation behavior of SH waves in a composite comprising of two different kinds of PE layers alternatively [10]. Considering the interface effect, Fang et al. studied the size-dependent dispersion relation of SH waves in a nano-sized PE/PM composite cylinder [11].

A common feature of the abovementioned papers [5–8,4,9–11] is that the interfaces studied therein are all perfectly bonded. However, as is known, the interfaces of practical PE/PM composite structures may become damaged or imperfect after a long time of service under harsh circumstances [12,13]. Employing the traditional linear spring model (LSM) to formulate the imperfect interfaces, Huang et al. [14], Nie et al. [15] and Yuan et al. [16] investigated the effect of interfacial imperfection on the dispersion behavior of SH waves. In their studies, only the interfacial mechanical imperfection was considered. Nevertheless, just like mechanical imperfection, electric and magnetic imperfections may also occur on the interface. When the electric and magnetic imperfections are also taken into consideration, the generalized

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linear spring model (GLSM) is then adopted to formulate the constitutive relation of the imperfect interface [17,18]. Based on the GLSM, Li and Lee [19], Sun et al. [20], and Kuo and Yu [21] studied the effects of magnetic, electric and mechanical imperfections on SH waves propagating along arc-shaped interfaces. Li et al. [22], and Jin and Li [23] surveyed the propagation behaviors of thickness-twist waves in inhomogeneous PE plates with interfacial electric and mechanical imperfections. Otero et al. analyzed the dispersion relations of interfacial waves between two half planes with a mechanically, electrically and/or magnetically imperfect interface [24,25].

When using GLSM to formulate the constitutive relation of an imperfect interface, it is generally assumed that the mechanical, electric and magnetic imperfections occur and exist independently [17–25]. However, this seems to be against the multi-physic essence of PE/PM composites [26,27]. As is known, MEE coupling is a prominent feature of PE/PM composites. If there are mechanical, electric and magnetic imperfections on the interfaces, they will be also inter-coupled together [26,27]. In the recent work of Li et al., the interfacial imperfection coupling models have been proposed for some different multiferroic composites, and the effects of such inter-couplings on fracture behaviors have also been revealed [26–28]. Except for fracture behavior, the SAW propagation may also be affected by such inter-couplings, and this still remains unexplored. In the present paper, we continue to study the propagation behavior of SH waves in a bi-layered multiferroic cylinder that contains a magneto–electro-mechanically imperfect interface. The interfacial imperfection coupling model [28] is employed to derive the dispersion equations, which are then numerically solved to reveal the effects of interfacial imperfection inter-couplings on the phase velocity. The obtained results and conclusions can serve as references for the property evaluation and optimal design of multiferroic composites.

2. Problem formulation

Shown in Fig. 1 is the cross section of a multiferroic composite that contains an imperfect interface. The composite is comprised of a central ferromagnetic (FM) cylinder with radius r_1 and an outer ferroelectric (FE) layer with outer radius r_2. Assume that the composite is polarized along the cylindrical axis. Then, it is transversely isotropic, and the cross section is the isotropic plane.

There are SH waves propagating along the interface. In this case, the composite is under axial shear deformation. The constitutive relations and governing equations can be stated as

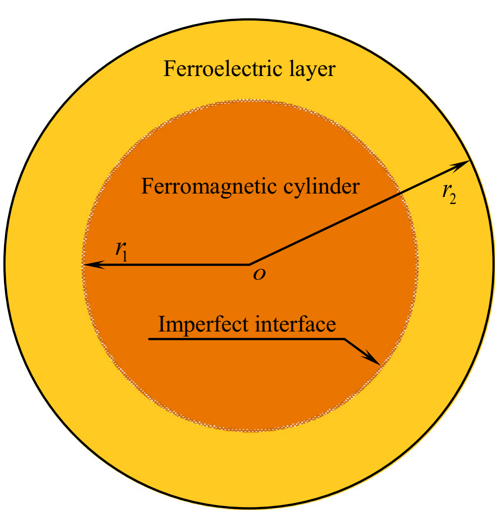

Fig. 1. The cross section of a multiferroic cylinder containing an imperfect interface.

$$\begin{pmatrix}\tau_{sz}^{(j)}\\ D_s^{(j)}\\ B_s^{(j)}\end{pmatrix} = r^{-\delta_{s\theta}} \begin{bmatrix} c_{44}^{(j)} & \delta_{2j}e_{15}^{(j)} & \delta_{1j}h_{15}^{(j)}\\ \delta_{2j}e_{15}^{(j)} & -\varepsilon_{11}^{(j)} & 0\\ \delta_{1j}h_{15}^{(j)} & 0 & -\mu_{11}^{(j)}\end{bmatrix}\begin{pmatrix} w_{j,s}\\ \phi_{j,s}\\ \varphi_{j,s}\end{pmatrix} \tag{1}$$

$$\left.\begin{array}{l} c_{44}^{(j)}\nabla^2 w_j + \delta_{1j}h_{15}^{(1)}\nabla^2\varphi_1 + \delta_{2j}e_{15}^{(2)}\nabla^2\phi_2 = \rho_j\ddot{w}_j\\ \delta_{1j}\left[h_{15}^{(1)}\nabla^2 w_1 - \mu_{11}^{(1)}\nabla^2\varphi_1\right] + \delta_{2j}[e_{15}^{(2)}\nabla^2 w_2 - \varepsilon_{11}^{(2)}\nabla^2\phi_2] = 0\\ \delta_{1j}\nabla^2\phi_1 + \delta_{2j}\nabla^2\varphi_2 = 0 \end{array}\right\} \tag{2}$$

where ∇^2 is the 2D Laplacian operator. $j = 1,\ 2$ and $s = r,\ \theta$. Hereafter, the quantities of the inner cylinder and the outer layer are marked by subscripts/superscripts 1 and 2, respectively. δ_{1j}, δ_{2j} and $\delta_{s\theta}$ are Kronecker delta functions, being 1 if the two subscripts are identical and 0 otherwise. τ, D and B are, in turn, the anti-plane stress, in-plane electric displacement and magnetic induction. w, ϕ and φ are the anti-plane mechanical displacement, in-plane electric potential and magnetic potential, respectively. c_{44}, e_{15}, h_{15}, ε_{11} and μ_{11} are the shear modulus, PE coefficient, PM coefficient, dielectric coefficient and magnetic permeability, respectively.

It is seen from Eq. (1) that both the dielectric coefficient of the FM cylinder and the magnetic permeability of the FE layer are taken into consideration. Therefore, the FM cylinder is at the same time a dielectric medium, and the FE layer is meanwhile a magnetic medium.

To solve the problem, one needs to specify the boundary conditions on the outer surface and the continuity conditions across the interface. Assume that the outer surface is mechanically traction-free. Then, the mechanical boundary condition is

$$\tau_r^{(2)}(r_2, \theta, t) = 0 \tag{3}$$

When the magneto-electric field in the free space is neglected [20], two cases of magneto-electric boundary conditions are considered here. In the first case, the outer surface is regarded as electrically open and magnetically shorted (EOMS), i.e.,

$$D_r^{(2)}(r_2, \theta, t) = 0; \quad B_r^{(2)}(r_2, \theta, t) = 0, \quad \text{(The EOMS case)} \tag{4}$$

In the second case, the outer surface is electrically shorted and magnetically open (ESMO), i.e.,

$$\phi_2(r_2, \theta, t) = 0; \quad \varphi_2(r_2, \theta, t) = 0, \quad \text{(The ESMO case)} \tag{5}$$

Next, we employ the interfacial imperfection coupling model (IICM) to formulate the constitutive behavior of the magneto–electro-mechanically imperfect interface [28]. According to IICM, the generalized stress $\boldsymbol{\tau}_r$ is continuous but the generalized displacement $\mathbf{w}$ is discontinuous across the interface. The former is proportional to the jump of the latter.

$$\boldsymbol{\tau}_r^{(1)}(r_1, \theta, t) = \boldsymbol{\tau}_r^{(2)}(r_1, \theta, t) \tag{6}$$

$$\boldsymbol{\tau}_r^{(1)}(r_1, \theta, t) = \boldsymbol{\alpha}[\mathbf{w}_2(r_1, \theta, t) - \mathbf{w}_1(r_1, \theta, t)] \tag{7}$$

where $\boldsymbol{\tau}_r^{(j)} = \left\{ \tau_{rz}^{(j)} \quad D_r^{(j)} \quad B_r^{(j)} \right\}^{\mathrm{T}}$ and $\mathbf{w}_j = \left\{ w_j \quad \phi_j \quad \varphi_j \right\}^{\mathrm{T}}$ $(j = 1, 2)$.

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 c_{44} & \alpha_4 e_{15} & \alpha_5 h_{15}\\ \alpha_4 e_{15} & -\alpha_2\varepsilon_{11} & -\alpha_6 d_{11}\\ \alpha_5 h_{15} & -\alpha_6 d_{11} & -\alpha_3\mu_{11}\end{bmatrix} \tag{8}$$

In Eq. (8), c_{44}, e_{15}, h_{15}, ε_{11}, μ_{11} and d_{11} are reference magnetoelectroelastic parameters. Specially, d_{11} is a reference magneto-electric parameter. $\alpha_1, \alpha_2, \ldots, \alpha_5$ and α_6 are the imperfect-interface parameters, which have the unified dimension of $[\text{length}]^{-1}$. α_1, α_2 and α_3 are interface parameters representing mechanical, electric and magnetic imperfections, respectively. When α_1, α_2 and α_3 have zero value, the interface is mechanically debonded and meanwhile electrically and magnetically impermeable. If α_1, α_2 and α_3 have enough