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Incorporation of a spatial source distribution and a spatial sensor sensitivity in a laser ultrasound propagation model using a streamlined Huygens' principle

Jernej Laloš *, Aleš Babnik, Janez Možina, Tomaž Požar

University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, 1000 Ljubljana, Slovenia

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ABSTRACT

The near-field, surface-displacement waveforms in plates are modeled using interwoven concepts of Green's function formalism and streamlined Huygens' principle. Green's functions resemble the building blocks of the sought displacement waveform, superimposed and weighted according to the simplified distribution. The approach incorporates an arbitrary circular spatial source distribution and an arbitrary circular spatial sensitivity in the area probed by the sensor. The displacement histories for uniform, Gaussian and annular normal-force source distributions and the uniform spatial sensor sensitivity are calculated, and the corresponding weight distributions are compared. To demonstrate the applicability of the developed scheme, measurements of laser ultrasound induced solely by the radiation pressure are compared with the calculated waveforms. The ultrasound is induced by laser pulse reflection from the mirror-surface of a glass plate. The measurements show excellent agreement not only with respect to various wave-arrivals but also in the shape of each arrival. Their shape depends on the beam profile of the excitation laser pulse and its corresponding spatial normal-force distribution.

1. Introduction

Ultrasound-propagation simulations in solids are indispensable in laser ultrasonics [1–5], optodynamics [1,6], optical pump–probe experiments [7,8], studies and applications of acoustic emission [9], non-destructive testing [10], seismology [11–15] and in metrology for sensor calibration [16–18].

Wave-propagation dynamics is described by a set of differential equations, initial and boundary conditions, which, in order to obtain a waveform solution, can be solved in a number of different ways: analytically, using a finite element method (FEM), a finite difference method (FDM), or Green's function formalism. Although the analytical solution is the most desirable, it is often nearly impossible to derive it due to the intertwined complexity of the wave equations, initial and boundary conditions.

FEM and FDM are valid methods for solving such a set of equations, especially for free-form geometries. They offer only whole numerical solutions for each specific instance and have to be entirely repeated for even a slight change in the geometry of the problem. Their solutions are bandwidth limited, depending on the density of the modeling mesh. Sharp changes in thus-calculated solutions are often accompanied by residual numerical oscillations that appear as unwanted artifacts in the waveform, bearing no physical meaning [19]. For this reason, the modeling of laser ultrasound often requires a dense mesh in order to distinguish different, sharp wave-arrivals. This requires high-frequency solutions, which, in turn, require large computing power to obtain them in a reasonable amount of time.

Green's function formalism, when used appropriately, enables modeling of different wave sources and their combinations without any major change to the model. For example, it is possible to model light-pressure-induced waves [20], thermoelastic waves and ablation-induced waves that are all present in laser ultrasonics [1,2,4]. Using this method, it is also possible for a composite waveform solution to be broken down to individual waves where each can be isolated in order to discern its propagation. Once Green's functions have been calculated or their band-limited counterparts obtained experimentally [19,21], this approach allows for significant changes in source-sensor selection without the need to repeat the entire equation-solving process. In another contrast with the FEM and FDM, by means of a deconvolution [22,23], Green's function formalism can be used to solve a backward problem – finding an unknown input signal from the known output.

The model, described here, is based on the interwoven concepts of Green's function formalism and the statistically streamlined

* Corresponding author.
E-mail address: jernej.lalos@fs.uni-lj.si (J. Laloš).

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Huygens' principle. It accounts for the real extents of the spatial source distributions and sensor sensitivity distributions. Huygens' principle has been used in waveform modeling before [24–26], predominantly to describe wide sources, while not simultaneously including sensor averaging.

The basic constituents of this computational approach are the direct time- and space-domain Green's functions as opposed to the transformed, temporal- and spatial-frequency-domain Green's functions [27]. To obtain either of them, the elastodynamic partial differential equation are converted to ordinary differential equations with integral transform techniques. The time domain is transformed into the temporal-frequency domain with either the one-sided Laplace or the Fourier transforms while a suitable choice for a radially-symmetric source is the Henkel transform which converts a space domain into a spatial-frequency domain [28,29].

The most demanding step in obtaining the direct time- and space-domain Green's functions is the inversion from the spatial-frequency domain to the direct space domain. The final results can rarely be given as closed-form expressions [30], thus careful numerical integration avoiding numerous singularities has to be performed [31].

The temporal dependence of the source is included in our modeling as a convolution of a time-domain Green's function with a suitable temporal profile of the source. Since a convolution in the time-domain corresponds to the multiplication in frequency-domain, the frequency-domain Green's function [27] can be taken as the starting building block of our construction scheme, thus avoiding one unnecessary step in performing the temporal convolution using a numerical routine based on the fast Fourier transform. One might be as well tempted to perform the same operation by including the radially-symmetric spatial distribution already in the space-frequency domain. Even though this step can be realized in some specific cases [28,29], it does not generally simplify the computation but rather makes it even more demanding by possibly introducing additional singularities into the inversion integrals.

Here, we provide a detailed step-by-step procedure. We describe how to construct an area-to-area (AA) waveform model from a much simpler point-to-point (PP) model with gradual overcoming of its limitations by incorporating the real geometric arrangements of both the source and the sensor, by means of a set of statistically weighted Green's functions.

A plate is chosen as it is one of the simplest and most frequently encountered geometric shapes, with practical scientific and engineering applications, while the ultrasound is chosen to be induced by circularly symmetric laser pulses. Out-of-plane displacement waveforms are commonly measured on either of its surfaces therefore, to facilitate their comparison and practical usefulness, the time-domain laser-pulse-induced surface waveforms are simulated.

To demonstrate the flexibility of the method, three distribution combinations and their corresponding weight functions are presented; they all incorporate a uniform sensor sensitivity distribution and a uniform (top-hat) [20], a Gaussian and an annular (ring) [20,26,32,33] spatial source distribution.

Such a model enables a more accurate representation of high-frequency waves with wavelengths shorter than the spatial extent of the source and the sensor, while Green's functions inherently provide a well-understood physical interpretation of the wave propagation theory.

Measurements of the light-pressure-induced ultrasound have been carried out using a piezoelectric sensor and two lasers, one with a uniform and the other with a Gaussian beam profile. To illustrate the viability of the AA model, a comparison of the measurements with the AA model and the simpler PP model is examined and evaluated in detail.

2. Development of the model

The main physical outlines of the problem are as follows: the transient source acts on the top surface of a homogeneous and isotropic plane-parallel plate and induces transient mechanical waves that propagate through it, i.e. ultrasound. Its surface waveforms, time histories of surface displacements, are of interest, since they are most commonly measured. The waveforms under consideration are comprised of waves that travel either directly from the source or are reflected, however many times, from the plate's surfaces but, at the same time, are not reflected from its sides. The model is thus intended for use in time-frames in which the plate can be considered infinitely large and in cases in which waveforms are measured outside the impact area, since this is a practical and quite common measuring arrangement.

2.1. Existing point-to-point model

Sound propagation in solids is described by a system of wave equations, their initial and boundary conditions, the solutions of which, for a δ-function input component, are Green's functions $g(\boldsymbol{v}_0, \boldsymbol{w}_0, t)$. They can be thought of as material transfer functions, which transform an input signal $f_\delta(\boldsymbol{v}_0, t) = \delta(\boldsymbol{v} - \boldsymbol{v}_0)\delta(t)$ at a point $\boldsymbol{v}_0$ into an output signal $u_\delta(\boldsymbol{w}_0, t)$ at a point $\boldsymbol{w}_0$. The geometric arrangement is shown in Fig. 1(a). Green's functions are highly specific, depending on the elastic and geometric properties of each material as well as the relative positions of the impulse input and waveform output locations. It is understood that, while not explicitly stated in the notation, Green's functions depend on the elastic constants of the plate along with its thickness.

Because of the δ-function input condition, the direct physical use of the Green's functions is somewhat limited to such cases where said point-to-point approximation applies. Due to their algebraic linearity and temporal invariability, this limitation can be overcome, in the temporal dimension at least, by performing a time convolution of a point-to-point Green's function $g_{\mathrm{PP}}(\boldsymbol{v}_0, \boldsymbol{w}_0, t)$ with a temporal part of the point-source impulse distribution. The latter is assumed to have separable temporal and spatial dependencies and can be written as:

$$f_P(\boldsymbol{v}_0, t) = J_0 s(t)\delta(\boldsymbol{v} - \boldsymbol{v}_0), \tag{1}$$

where J_0 is the magnitude of the source impulse equal to the linear momentum transfer to the plate at a point $\boldsymbol{v}_0$ and $s(t)$ is its normalized temporal distribution. In this point-to-point (PP) model, the displacement waveform at a point $\boldsymbol{w}_0$ due to a point-source is obtained:

$$u_{\mathrm{PP}}(\boldsymbol{w}_0, t) = J_0 \int_{-\infty}^{\infty} g_{\mathrm{PP}}(\boldsymbol{v}_0, \boldsymbol{w}_0, \tau) s(t - \tau) d\tau. \tag{2}$$

In reality, however, the source impulse usually does not act on an infinitesimally small point on the surface but on a certain macroscopic area. It is for this reason that the spatial limitation of the PP model must be addressed further in the expanded mathematical models.

2.2. Incorporation of the spatial source distribution

In this model expansion, a statistically streamlined Huygens' principle is used to calculate, at a certain detection point $\boldsymbol{w}_0$, a displacement waveform $u_{\mathrm{AP}}(\boldsymbol{w}_0, t)$ that was induced by a macroscopic, circularly symmetric source impulse $f_A(\boldsymbol{v}, t)$ impacting a circular area of random points $\boldsymbol{v}$ with a radius of r_{F0} and centered at $\boldsymbol{v}_0$. The corresponding geometry is shown in Fig. 1(b). A distinction has to be made between cases where points $\boldsymbol{v}$ and $\boldsymbol{w}_0$ are coplanar and where they are not. If point $\boldsymbol{w}_0$ does not lie on the impact sur-