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A quantitative method for evaluating numerical simulation accuracy of time-transient Lamb wave propagation with its applications to selecting appropriate element size and time step

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ABSTRACT

Lamb wave technique has been widely used in non-destructive evaluation (NDE) and structural health monitoring (SHM). However, due to the multi-mode characteristics and dispersive nature, Lamb wave propagation behavior is much more complex than that of bulk waves. Numerous numerical simulations on Lamb wave propagation have been conducted to study its physical principles. However, few quantitative studies on evaluating the accuracy of these numerical simulations were reported. In this paper, a method based on cross correlation analysis for quantitatively evaluating the simulation accuracy of time-transient Lamb waves propagation is proposed. Two kinds of error, affecting the position and shape accuracies are firstly identified. Consequently, two quantitative indices, i.e., the GVE (group velocity error) and MACCC (maximum absolute value of cross correlation coefficient) derived from cross correlation analysis between a simulated signal and a reference waveform, are proposed to assess the position and shape errors of the simulated signal. In this way, the simulation accuracy on the position and shape is quantitatively evaluated. In order to apply this proposed method to select appropriate element size and time step, a specialized 2D-FEM program combined with the proposed method is developed. Then, the proper element size considering different element types and time step considering different time integration schemes are selected. These results proved that the proposed method is feasible and effective, and can be used as an efficient tool for quantitatively evaluating and verifying the simulation accuracy of time-transient Lamb wave propagation.

1. Introduction

Compared with point-to-point inspection using traditional ultrasonic bulk waves, Lamb wave technique provides a cost-effective inspection method in non-destructive evaluation (NDE) and structural health monitoring (SHM), as Lamb wave is able to travel a very long distance with little energy loss and can be used to interrogate physically inaccessible areas of structures and components. However, due to the multi-mode characteristics and dispersive nature, Lamb wave propagation behavior is much more complex than that of bulk waves. Understanding physical principles of Lamb wave propagation is indispensable for fully exploiting the advantages of Lamb wave technique and thus beneficial to its applications in NDE and SHM.

Analytical approaches [1–4] have been used to resolve Lamb waves propagation problems. In these references, explicit expressions, known as the Rayleigh–Lamb frequency equations, were derived and dispersion curves, a fundamental way of describing Lamb wave propagation in a specified structure, could be easily plotted based on these equations. Giurgiutiu [5,6] published a closed equation of time-transient Lamb wave displacement response under the excitation of piezoelectric wafer active sensor (PWAS). Although analytical approaches are precise, they are only applicable to simple and regular structures.

Besides analytical approaches, a great number of numerical simulation methods [7–10] have been employed to study the Lamb wave propagation. Willberg et al. [10] reviewed the

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state-of-the-art numerical simulation methods used in guided wave-based SHM. These approaches include finite element analysis [11–24], finite difference equations [25–27], finite strip element [28,29], boundary element method [30,31,56], semi-analytical finite element analysis [32–40], global matrix methods [41], spectral element approaches [42–48], mass-spring lattice models [49,50], the local interaction simulation approach (LISA) [51–53], finite cell method [54], and the spectral cell method [55]. The use of these numerical methods providing diverse solutions to time-transient Lamb wave response under certain conditions, is beneficial to understanding the fundamental physical principles of Lamb wave propagation, and paves the way for expanding the applications of Lamb wave technique in various industries.

A successful numerical solution to a practical problem should be composed of not only numerical method itself but also measurement strategy for the evaluation of simulation accuracy. However, according to our literature review, considerable attention has been focused on the simulation using different numerical methods, whereas accuracy evaluation has received little attention. Some authors [17] just pointed out that their simulation results were comparable with previous published studies; some authors [19,40,43] only evaluated the simulation accuracy by checking the time of flight of the simulated wave packets visually. And in most of the numerical simulation research work, the simulation accuracy of time-transient Lamb wave propagation is evaluated as follows. First, simulation waveform is superimposed with the corresponding analytical waveform [16,18,20,33,34] or experimental signals [8,24,27] in one figure. Then the simulation accuracy is qualitatively verified by visually checking and comparing the difference between them. At present, few quantitative analysis on evaluating simulation accuracy has been reported. Willberg et al. [44] and Duczek et al. [48] developed and compared higher order finite element schemes for simulating Lamb wave propagation and presented a convergence indicator to quantify numerical accuracy and performance. In their studies, the convergence indicator is computed from Hilbert transform and is actually used for quantifying the group velocity accuracy (namely the position accuracy in this paper). However, the shape accuracy was not considered in their investigations. In this paper, a quantitative method based on cross correlation analysis is proposed. Cross correlation analysis is used to measure the similarity between two signals at different times. Although cross correlation analysis has been widely used in many fields [57–59], it is the first time to be applied to quantitatively study the simulation accuracy evaluation of time-transient Lamb waves propagation. In the previous studies [60,61], wavelet analysis has been used to compute the group velocity. However, it is quite complicated to evaluate the similarity between two signals using wavelet analysis. Therefore, compared with wavelet analysis, our proposed method using cross correlation analysis is quite efficient and easy to implement, as two indicators to evaluate numerical simulation accuracies including the position accuracy and shape accuracy can be calculated simultaneously.

Two kinds of error, influencing the position and shape accuracies of simulated waveform are firstly identified. Consequently, two quantitative indices, i.e., the GVE (group velocity error) and MACCC (maximum absolute value of cross correlation coefficient) derived from cross correlation analysis between simulated signal and reference waveform are thus proposed to assess the position and shape errors of the simulated signal. As the proposed method is based on cross correlation analysis, a reference waveform or signal derived from the corresponding analytical or experimental study is a prerequisite.

Among the above numerical approaches, FEM is the most popular and widely used technique, as it is very straightforward, easy-to-learn and convenient to select general simulation platform (e.g. commercial FEM software). However, in order to achieve accurate FEM simulation, selecting appropriate parameters of element size and time step is of great importance. In this paper, the proposed quantitative simulation accuracy evaluation method is applied to selecting proper element size regarding different element types and time step regarding different time integration schemes.

Up to now, there is no FEM software offering simulation accuracy evaluation function, let alone quantitative accuracy evaluation function. In this paper, an innovative FEM program integrating simulation accuracy quantitative evaluation function for 2D time-transient Lamb wave propagation is developed. This software is designed to provide a platform for applying the proposed quantitative simulation accuracy evaluation method to selecting proper parameters of element size and time step.

Regarding the selection of the parameter of element size, various researchers have their own choices. Alleyne and Cawley [62] reported that using quadrilateral elements, substantially more than the threshold of 8 elements per wavelength is a good limit for accurate modeling of wave propagation problem; Xu [23] used an element size corresponding to 10 elements per wavelength; Moser [20], Gresil [18], Shen and Giurgiutiu [63] and Wan [64] employed an element size equal to 1/20 of the shortest wavelength of interest. In these studies, the authors just used one kind of element type, and did not quantitatively verify the simulation accuracy. In general, accurate numerical simulation can be achieved by reducing element size. However, small element size results in much computation time. Therefore, selecting an appropriate element size is still an open problem. Recently, Willberg et al. [44] and Duczek et al. [48] studied the optimal element size for high-order finite element schemes. In this paper, the dependence of GVE and MACCC on element size considering different element types is studied and only low-order element types are considered. An appropriate element size regarding element types is selected when both the position and shape accuracies reach a high level.

Just like the parameter of element size, the selection of an appropriate time step is also still worth discussing. Bathe [65] proposed that the maximum time step satisfying stability for an explicit time integration method is given by $\Delta l/c$, where Δl is the element size and c refers to the wave speed of the fastest wave mode. Moser [20], Gresil [18], Shen and Giurgiutiu [63] and Wan [64] used the expression $\Delta t = 1/(20 * f_{\max})$ to select time step, where Δt is the time step and $f_{\max}$ refers to the highest frequency of Lamb wave mode. In these studies, the authors just used one kind of time integration scheme, and did not quantitatively validate the simulation accuracy. In this paper, the dependence of GVE and MACCC on time step regarding different time integration schemes is studied. An appropriate time step regarding time integration schemes is selected when both the position and shape accuracies approach a high level.

The rest of this paper is organized as follows. Section 2 introduces the basic theories including cross correlation analysis, analytical model of dispersion curves of Lamb waves, theoretical analysis of time-transient Lamb wave propagation under the excitation of PWAS, and finite element method for time-transient Lamb propagation simulation. In Section 3, a quantitative method for evaluating the simulation accuracy is proposed and an innovative software providing both finite element simulation function and accuracy quantitative evaluation function for 2D time-transient Lamb wave propagation is developed. In Section 4, the applications to selecting proper element size regarding different element types and time step regarding different time integration schemes are presented. Conclusions are drawn in section 5.