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Orbital-type trapping of elastic Lamb waves

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ABSTRACT

The interaction of laser-generated Lamb waves propagating in a plate with a sharp-angle conical hole was studied experimentally and numerically. Part of the energy of the incident wave is trapped within the conic area in two ways: the antisymmetric Lamb wave orbiting the center of the hole and the wave localized at the acute edge. Parameters and conditions for optimal conversion of the incident wave into the trapped modes were studied in this work. Experiments were performed using the laser stroboscopic shearography technique, which delivers the time evolution of the acoustic field in the whole area of interest. The effect of trapping can be used for efficient damping, similar to the one-dimensional acoustical black hole effect.

1. Introduction

The Acoustic Black Hole (ABH) effect has been widely studied as an application for an efficient damper of flexural waves in plates or structural vibration of flexural type. It utilizes the slowing down of the flexural plate wave as the plate becomes thinner forming a sharp wedge. It was shown [1] that if the thickness h is dependent on the lateral coordinate x and tends to zero obeying the power law $h = \varepsilon x^m$ with $m \geqslant 2$, the velocity of the flexural wave decreases with $x \to 0$ fast enough that it never reaches the point $x = 0$. The wave gets "trapped" in space with both phase and group velocities being infinitesimally small, and is not reflected. Krylov et al. [2–5] investigated the one-dimensional power-law wedges and two-dimensional ABH (power-law profiled pits) theoretically and experimentally. It was proposed to introduce an absorbing layer near the edge, where the wave is trapped, to enhance the damping. The power law for the thickness requires the sample to be very thin in the tip area, which makes fabrication of such wedges difficult, and in practice can lead to deformation or damage of the edge. To enhance attenuation of the flexural wave, plates containing ensembles of several two-dimensional ABH were studied [6]. Sound propagating in the air can also be silenced by means of the ABH effect in the graded sonic crystals [7]. In addition to the thickness variation, Cuenca and Pelat [8] proposed to use temperature gradient to modulate the material stiffness. In acoustic gradient index materials many phenomena like refraction, convergence and collimation of wave can be realized. Lin [9] has designed a 2D gradient-index phononic crystal structure to control the propagating direction realizing the 'acoustic mirage' effect. Acoustic lens based on the gradient-index material was realized by Climentae [10]. Gradient-index acoustic absorber with axially symmetric variation of the sound velocity has been studied numerically [11,12]. In the proposed scheme the incident longitudinal wave refracts toward the absorbing area [11] or can be trapped at the circular orbit [12]. For bulk acoustic waves it is difficult to realize in practice the prescribed spatial variation of the sound speed, whereas the velocity of the Lamb waves can be easily controlled. We have made use of this fact to study experimentally the effect of the wave capturing in an axially-symmetric trap.

2. Theoretical background

Compared to the bulk acoustic waves, the waves in the plates exhibit significantly stronger variations of the phase velocity; the acoustical refractive index for a plate wave can be varied in a vast range from 1 to practically infinite values by tuning the thickness of the conveying plate. Therefore the two-dimensional ABH effect is more apparent and simple for the plate waves. In this work we study trapping of the flexural plate wave in the axially symmetric ABH.

Consider a plane antisymmetric Lamb wave of A_0 type, which propagates in a plate with a radial thickness variation $h = h(r)$. In the geometrical acoustic (GA) approximation, the condition for existence of a stationary trajectory of the wave orbiting the axis of symmetry (*i.e.* the trapped wave) lying at the radius r_0 can be written as following:

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$$\left.\frac{\partial V(r)}{\partial r}\right|_{r_0} = \frac{V(r_0)}{r_0} \quad \text{or} \quad \left.\frac{\partial \Omega(r)}{\partial r}\right|_{r_0} = 0 \tag{1}$$

where V is the phase velocity of the wave, $\Omega = V/r$.

For sufficiently small frequencies ω, condition (1) can be satisfied in a parabolic pit with $h(r) = \varepsilon r^2$. If $\omega \ll V/h$, the phase velocity of the A_0 Lamb wave has the following asymptotic: $V = \sqrt{h(r)\omega V_P/\sqrt{3}}$ or $V(r) = r\sqrt{\omega\varepsilon V_P/\sqrt{3}}$ where V_P = const, and (1) holds for all ω, ε and r. Note, that two contradictory conditions must be satisfied simultaneously for this consideration to be valid, the GA condition $\partial V/\partial r \ll \omega$ and the low-frequency limit. This sets the lower limit for the radius r: $r^2 \gg \varepsilon^{-2}$ and also restricts the frequency range: $\varepsilon V_P \ll \omega \ll V_P/(\varepsilon r^2)$. For wider frequency range the thickness function of a more complex form can be found numerically, whereas the parabolic indentation can still exhibit a relatively wide-front trapping.

In this work we consider trapping at the conical hole with thickness $h(r) = a(r - R_0)$, $R_1 > r \geqslant R_0$ and radius R_0 and R_1 on the opposite sides of the plate, as shown in Fig. 1. Such geometry can support not only the stationary Lamb waves satisfying Eq. (1), but also the antisymmetric waves guided by the acute edge of the hole. Those waves are in many features similar to the localized and nondispersive wedge waves localized near the tip of the straight elastic wedge [13], but in a circular wedge they are not totally localized at the tip. In contrast to the wedges with quadratic or higher power law thickness [14,15], their phase velocity remains nonzero for the case of conical hole. Thus as illustrated in Fig. 1 the conic hole offers two ways for the wave trapping: stationary orbital wave of the Lamb type, and conversion into the edge-localized waves (ELW) at the tip.

In order to be able to use the GA approximation in the most part of the cone, the parameter a must be sufficiently small. Below we consider the hole with R_0 = 1 mm, R_1 = 5 mm, and the plate thickness h = 0.5 mm (a = 0.125 mm^{-1}). The plate material is aluminum alloy with longitudinal and transversal waves velocities 6153.4 m/s and 3099.6 m/s respectively. For those parameters the GA is applicable for $r \geqslant R_0 + 0.5mm$ at a characteristic frequency of 1 MHz. Local phase velocities of the A_0 Lamb mode were calculated for different frequencies from the Rayleigh–Lamb equation as functions of the radial coordinate r. Corresponding angular velocities $\Omega(r)$ are shown in Fig. 2 for the frequencies 0.5 MHz, 1 MHz, 2 MHz and 4 MHz.

Stationary radius r_0 determined by Eq. (1) is shown for the 1 MHz curve in Fig. 2. It decreases slowly with growing frequency, which has a little effect on the wave trapping.

Refraction of the beam within the conic area conserves the tangential component of the wave vector:

$$\frac{1}{V(r)}\sin(\theta) = \text{const} = \frac{\sin(\theta_i)}{V_0} \tag{2}$$

where θ stands for the directional angle between the beam and the radial vector $\vec{\mathbf{r}}$, θ_i is its value at the incidence, and V_0 is the velocity of the Lamb wave outside the cone. For the wave trapped at the stationary orbit at r_0, the incidence angle θ_s and the stationary velocity $V(r_0)$ are related as $\sin(\theta_s) = V_0/V(r_0)$. Beam which enter the cone with $\theta < \theta_s$, reaches r_0 with $\theta < 90°$ and therefore it satisfies

$$\frac{\partial V}{\partial r} > \frac{V}{r} \quad \text{or} \quad \frac{\partial}{\partial r}\Omega < 0,\ r \leqslant r_0 \tag{3}$$

and propagates toward the edge of the hole at $r = R_0$. Along its propagation direction the angle θ tends to zero as

$$\sin(\theta) = 1/n_a(r) \tag{4}$$

where acoustic refractive index is defined as $n_a(r) = V(r_0)/V(r)$. Near the edge n_a is very big, so the beam incidence is almost normal to the edge.

Efficient conversion of the Lamb wave into the wave propagating along the edge takes place if the radial components of their wave vectors match. Combining this statement with Eq. (2) delivers the following equation:

$$\frac{V_0}{\sin(\theta_i)} = V_\theta(R_1) = V_W \tag{5}$$

and $\theta_i < \theta_s$, where V_W is the phase velocity of the wave propagating along the wedge and V_θ is the tangential component of the velocity. Thus the conditions for the incident wave to be converted to ELW are:

$$\sin(\theta_i) = V_0/V_W; \quad \theta_i \leqslant \theta_s \tag{6}$$

Besides this mechanism of conversion, the stationary orbiting wave can be coupled directly to the ELW if its orbit lies sufficiently close to the edge, *i.e.* r_0–R_0 is smaller than the radial extension of the ELW. This type of wave coupling requires the equality of angular velocities of both waves:

$$R_0 V(r_0) = r_0 V_W \tag{7}$$

Note, that the waves propagating along the edge $r = R_0$ are not totally localized wedge waves that exist at the straight wedges [13]. They can be regarded as leaky waves [16], in the sense that their amplitude does not decay to zero away from the apex.

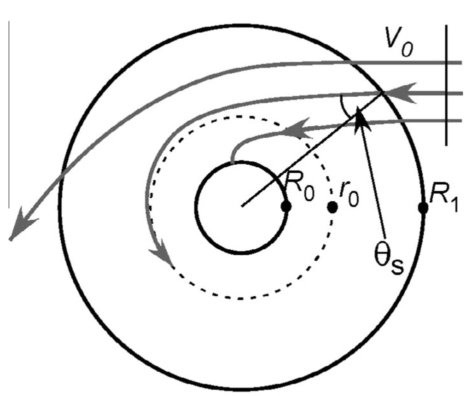

Fig. 1. Refraction of the A_0 Lamb wave within the conic hole with radii R_0 and R_1 on two sides of the plate.

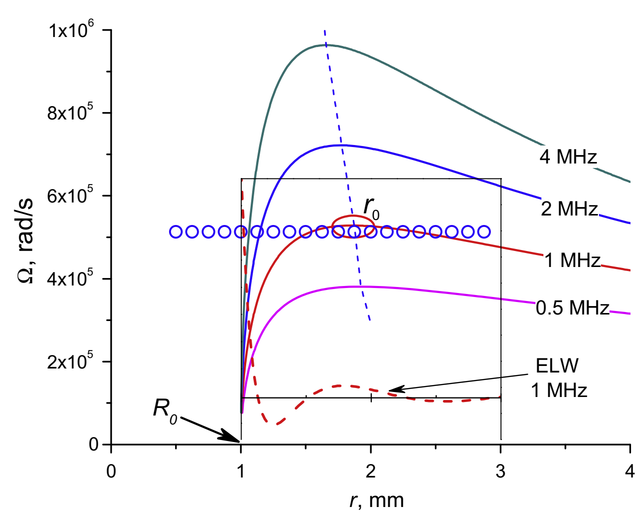

Fig. 2. $\Omega(r)$ for different frequencies in the cone specified in Fig. 1. Circles denote the V_W/R_0. Inset shows the radial distribution of the edge-localized wave (ELW) with 1 MHz frequency.