



# Multiple scattering of an acoustic wave from a network of cylindrical rods encased in a solid viscoelastic medium



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## ABSTRACT

In this paper, the multiple scattering of normally and obliquely incident acoustic waves from a network of cylindrical rods embedded in a viscoelastic medium is investigated. The cylinders can be either isotropic or transversely isotropic. This problem has applications in studying the interaction of ultrasonic waves with composite materials in ultrasonic nondestructive testing. By extending the classical theory of scattering from a single cylinder, the multiple scattering equations are derived incorporating all three kinds of longitudinal, horizontally polarized shear and vertically polarized shear waves in a viscoelastic matrix. The spatial angle of the incident wave is arbitrarily chosen and the backscattered waves can be found in any point of the space. In the proposed method, there are no restrictions on the number of cylinders and/or on the order of scattering. Therefore, the method is capable of handling any order of scattering from any number of cylinders. To experimentally validate the numerical results, a number of experiments are conducted on steel cylinders embedded in a polymeric matrix. The scattered waves from the cylinders are received and their corresponding form functions are obtained. By comparing the numerical and experimental results, a good agreement is observed.

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## 1. Introduction

When a target with specified dimensions is insonified by an incident elastic/acoustic wave, in addition to reflection, refraction and mode conversion, waves may also scatter from the target into the surrounding media. Upon the interaction of the incident wave with the elastic target, surface waves are generated around the periphery of the cylinders and upon constructive interference, they build up to the resonance modes of the target. By examining the resonance frequencies and resonance modes, valuable information regarding the characteristics (both geometry and material) of the target is obtained. This procedure is called *Resonance Acoustic Spectroscopy* (RAS). In RAS, a broad-band acoustic wave is used to excite the resonant modes of an immersed target. If the wave frequency coincides with one of the resonance frequencies of the target, the corresponding resonance mode of the immersed body is excited. Consequently, the body starts vibrating and energy is dissipated into the surrounding medium.

The scattering of ultrasonic waves from embedded targets can be used for gaining more information about the characteristics of these embedded targets. The backscattered waves contain valuable

information that can be used for characterization of the target. To investigate the scattering phenomenon in fiber-reinforced composite materials under the impact of ultrasonic waves, a number of points should be taken into account. Firstly, we usually face a network of cylinders (and not one single cylinder) encased in a solid matrix. Secondly, the matrix material surrounding the cylinders is usually a viscoelastic polymeric material whose attenuation cannot be neglected. Therefore, to accurately model the scattering phenomenon in fiber-reinforced composites, it is necessary to account for the viscoelastic properties of the matrix. Thirdly, if only normally incident waves are considered, limited cases can be studied. To have a broader picture of the scattering phenomenon, it is necessary to consider both normally and obliquely incident waves. Studying the obliquely incident waves makes it possible to get access to more information about the structure and/or material characteristics.

Several studies have been conducted by various researches to reveal the effects of surface waves on resonance phenomenon. In these investigations, it has been shown that when the frequency of the incident wave coincides with one of the natural frequencies of the cylinder, the surface waves interfere constructively and lead to the formation of standing waves around the cylinder. This is the basis of Resonance Scattering Theory (RST) which was firstly introduced by Flax et al. [1]. Talmant et al. [2] used RST for studying the

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scattering of acoustic waves from thin air-filled aluminium shells. To investigate the scattering of ultrasonic waves from anisotropic cylinders, de Billy conducted a number of experiments [3]. Employing the method of normal mode expansion, Honarvar and Sinclair [4] presented an analytical solution for the problem of scattering of compression waves from a transversely isotropic cylinder immersed in a fluid. Following the same procedure, Fan et al. [5] used the method of normal mode expansion to develop a new analytical model for scattering of ultrasonic waves from a transversely isotropic cylinder encased in an elastic matrix. Biwa et al. [6] presented a computational method for multiple scattering of transverse waves from a fiber-reinforced composite material. Jamali et al. [7] reported on the scattering of elastic waves from a cylinder made of functionally graded materials (FGM). The method was developed for two dimensional problems by employing the T-matrix method. In recent years, this field of research has attracted a great deal of interest due to new developments in the nondestructive evaluation (NDE) methods.

Lethuillier et al. [8] performed theoretical and experimental studies of acoustic wave scattering from a finite linear grating of elastic cylindrical shells. It has been observed that a resonant interaction occur at low frequencies where the shells are very close to each other. Le Bas et al. [9] showed that this phenomenon is also seen in the case of a triangular grating of immersed cylindrical shells. Robert et al. [10] considered multiple scattering by a finite number of closely-spaced cylindrical cavities (both empty and fluid-filled) embedded in an elastic matrix. They showed that, with the exception of a few narrow resonances, the single fluid-filled cavity resonances do not display the splitting phenomenon seen for immersed elastic scatterers.

Cai [11] presented a closed form solution for scattering of anti-plane shear waves by a layered circular elastic cylinder embedded in a solid elastic medium. The scattering of a plane acoustic wave by an infinite penetrable or impenetrable circular cylinder, parallel with another one, of acoustically small radius, is presented by Roumeliotis et al. [12]. The method of separation of variables, in conjunction with translational addition theorems for cylindrical wave functions, was used in their study. The eigenmodes of the time-reversal operator are studied for two elastic cylinders by Minonzo et al. [13]. The effects of multiple scattering and anisotropic scattering are considered in their study. The resonance scattering by a periodic infinite array of fluid-filled cylindrical cavities in an elastic matrix is studied by Robert et al. [14]. Chen et al. [15] studied the multiple scattering of sound waves from circular cylinders embedded in an elastic medium.

Hasheminejad and Alibakhshi [16] considered the fundamental problem of 2-D acoustic scattering. They treated the interaction of a plane compression sound wave with a cluster of two flexible fibers submerged in a boundless viscous fluid medium.

Chen et al. [17] studied the effect of water-wave force on four rigid cylinders insonified by a normally incidence wave. Sodagar et al. [18] used a mathematical model for the scattering of obliquely incident plane acoustic waves from a grating of immersed cylindrical shells and conducted a detailed study of the interaction of A-wave resonances originating from the shells.

In this paper, we first review the general case of scattering of a plane obliquely incident wave from a transversely isotropic cylinder embedded in a solid elastic matrix. We then, employ the Graf's Addition Theorem [19] and extend this method to properly formulate the case of multiple scattering. Furthermore, in order to take into account the effect of viscoelastic behavior of the matrix material, the required constitutive relations are incorporated into the formulation. Some examples are presented to show the applications of proposed method to different problems. The effects of the angle of incidence with respect to the cylinder axis are argued in an example.

To experimentally validate the numerical results of multiple scattering in viscoelastic and non-viscoelastic media, a number of experiments are conducted on polymeric blocks in which a number of parallel steel rods are embedded. With the aid of an ultrasonic probe, the wave scattered from the cylinder is measured and the corresponding form functions are plotted.

## 2. Mathematical model for scattering from parallel cylinders

It is assumed that a planar wave is incident on parallel cylinders at an arbitrary incidence angle. The incident wave angle is  $\alpha$  with respect to the horizontal axis and makes an angle of  $\beta$  with respect to the centerline of the cylinders. The geometrical configuration of the planar incident wave on two parallel cylinders encased in a solid matrix is shown in Fig. 1.

In order to develop the method for multiple scattering, firstly, the mathematical model for a single solid cylinder embedded in a solid matrix is described in brief.

Consider a plane wave traveling with frequency of  $\omega/2\pi$  through an infinite isotropic medium and incident at an angle  $\alpha$  on a long transversely isotropic cylinder of radius  $a$  (Fig. 2). The cylinder and matrix densities are denoted by  $\rho_c$  and  $\rho_m$ , respectively. According to Fig. 2, the coordinate is set in such a way that the z-axis coincides with the axis of the cylinder.

The displacement field within both the matrix and cylinder may be expressed by the following equation [4]:

$$\mathbf{u}(r, \theta, z) = \nabla\phi + \nabla \times (\chi\mathbf{e}_z) + a\nabla \times \nabla \times (\psi\mathbf{e}_z) \quad (1)$$

in which  $\phi$ ,  $\chi$  and  $\psi$  are scalar potential functions. In general,  $\phi$  corresponds to compression,  $\chi$  to transversely polarized shear, and  $\psi$  to axially polarized shear components. In the solid matrix, any of these potential functions could consist of an incident plane wave component and a scattered wave component. Considering the general form of governing equations of wave propagation based on the motion equations and assuming propagation of the wave with a frequency of  $\omega$ , we can find the general form of these potential functions. In what follows, indices 1 and 2 are attributed to cylinder and matrix parameters, respectively. In case of a compression incident wave, we may write:

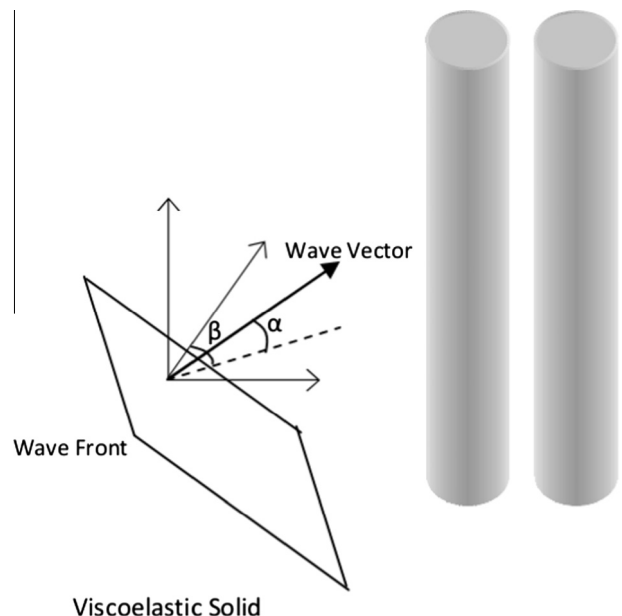


Fig. 1. Geometrical configuration of the plane incident wave on two parallel cylinders encased in a solid matrix.

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