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A Bayesian approach for high resolution imaging of small changes in multiple scattering media

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ABSTRACT

This paper introduces a Bayesian approach to achieve high-resolution imaging of sub-wavelength changes in the presence of multiple scattering. The approach is based on the minimization of a cost function defined by the decorrelations induced in the measured waveforms by the apparition of a local changes. Minimization is achieved via a Monte Carlo Markov Chain (MCMC) algorithm combined to an analytical model that computes the sensitivity kernel of the medium. In the inversion procedure, the parameters to infer represent the physics of the problem, such as the diffusivity in the medium and/or the geometrical features of the reflector (position and scattering cross-section). The method is successfully compared to the linear inversion approach initially proposed for the so-called **Locadiff** imaging method through several examples, both numerical and experimental.

1. Introduction

The study of wave propagation in multiple scattering media is of interest to many research domains at scales ranging from the nanometer (*e.g.* optic waves) to the kilometer (*e.g.* seismic waves). Typically multiple scattering occurs when the average propagation distance between two scattering occurrences, *a.k.a* the mean free path, ℓ, is smaller than the source-receiver distance, and in practice of the order of a few wavelengths. Since the presence of heterogeneities favors the scattering events, a material that is heterogenous at the scale of the wavelength is generally considered as multiply scattering. One of the main challenges currently is to image these media because the propagation of multiply scattered waves can be assimilated to a random walk between the heterogeneities of the medium [1]. Thus, information about the original direction is lost after only a few mean free paths and classical inspection methods such as those based on the arrival times of direct waves cannot be used. Attempts to image multiple scattering media were made in a wide scope of applications, such as diffuse optical tomography [2], medical imaging [3] nondestructive evaluation (NDE) [4,5], and even geophysics [6,7]. However, accurate imaging of small changes is still out of reach.

Recently, a deterministic method based on the random matrix approach allowed successful imaging of a reflector located about three mean free paths away from an ultrasonic array [8]. This method relies on the ability to distinguish between simple and multiple scattering via a singular value decomposition of the time reversal operator (DORT). However, when the scattered energy from the reflector nears that of the scatterers, information about the reflector may be hidden in the smallest singular values, in which case DORT-like methods may fail. Therefore the ability of this approach to detect reflectors that either have a small dimension compared to the wavelength or are located many scattering mean free paths away from the sources is still an on-going research.

Rather than attempting to filter out the multiple scattering, an interesting alternative is to take advantage of the fact that the scattered wave provides a broad and sensitive sampling of the medium, because the longer the time in the scattered wave, the more interactions with the reflector. This forms the basis of coda wave interferometry (CWI) characterization methods [1], where the term coda, originally used in Geophysics to name the late part of a seismogram [9], is now also used in the other fields of wave physics to denote the superposition of simple and multiple scattering. In Geophysics CWI has been used for more than 30 years, for instance to monitor structural changes in faults [10], volcanoes [11], landslides [12]. In NDE, applications are recent and concern mainly the detection of damage apparition and/or the monitoring of damage evolution [4,5,13].

These methods focus on detecting time-lapse changes in the propagation medium by comparing coda signals, and exhibit an advanced sensitivity due to the use of the scattered waves. However, they monitor the propagation medium as a whole which

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makes localization impossible, because the propagation trajectory of coda waves is unknown. However, without deterministic approaches, successful imaging of a multiple scattering medium requires both a reliable forward model and an appropriate inversion method. The imaging of local changes was made possible only recently, thanks to a technique named *Locadiff* [14,15]. In this technique, a sensitivity kernel of the medium is calculated by simulating the acoustic intensity of coda waves [16,17]. The sensitivity kernel is used to describe the propagation of the scattered waves in a statistical sense, and the changes induced by the change in the coda can be quantified by the product between the scattering cross-section of the change, γ_e, and the sensitivity kernel at its location. Inferring the position of the local change can be achieved by solving the associated inverse problem.

The inversion procedure relies on the minimization of a cost function where experimental data are compared to output data from a model [18,19]. Minimizing the cost function can be achieved in different ways. The first version of *Locadiff* [14] used the simplest approach which consists of a classical grid search. In consequence, the solution becomes intractable when the number of parameters is larger than 3 or 4 (i.e. several changes). More recently, the Gauss–Newton minimization method [20] was also tested. Due to its ability to solve multi-parameters problems, compared to a grid search this method showed important improvements to detect several structural changes. In the context of a geophysical study [7], its ability to locate simultaneously several changes at different places was demonstrated experimentally [7].

However, like any gradient-descent-like method, Gauss–Newton minimization approach may lack accuracy or even diverge when the algorithm is started far away from the global minimum. Meanwhile, a Bayesian approach can overcome these difficulties and provide an estimate of the parameters with very good accuracy. In this paper, we investigate the potential of minimizing the cost function in *Locadiff* with the Monte Carlo Markov Chain (MCMC) algorithm. MCMC methods have been used for decades in various research fields such as biology [21], economics [22], or neural sciences [23]. Recently their application have also been extended to NDE, thanks to efficient modeling tools: Zhang et al. used the Metropolis–Hastings algorithm to evaluate grain orientation in a weld [24], Moreau et al. used a Bayesian approach to reconstruct the geometry of 3D scatterers in waveguides [25].

This approach is used here for inverting parameters associated with the multiple scattering problem, *i.e.* the geometrical features of several changes, as well as the properties of the multiple scattering medium itself (Section 2). Numerical (Section 5) and experimental cases (Section 6) are investigated, and for comparison purposes results from the linear inversion method are also presented. First, the robustness of the approach is tested by simulating the acquisition of experimental data where the level of measurement error can be controlled (Section 5.1). Then, the accuracy of the approach is investigated on a more complicated case where 3 reflectors with different geometries are simulated (Section 5.2). Finally, an experimental example is presented where the method is used to image a hole drilled in a concrete block (Section 6). Based on these investigations, advantages of the method as well as current limitations are discussed, and possible improvements are suggested.

2. Bayesian procedure for solving the inverse problem

2.1. Cost function

In a multiple scattering medium, coda waves propagate along complex trajectories, each can be assimilated to a random-walk between scatters. The superposition of coda waves arriving at the sensor is recorded as the coda signal, which temporal waveform is a deterministic representation of the inner structure of the propagation medium. Although coda signals have noisy appearance, they do not fluctuate randomly like noise. Instead, they remain identical unless one or several changes occur in the medium, for instance a reflector or a local change of velocity. The local change (s) modify the propagation trajectory of coda waves, which translates into modifications of the temporal waveform in the coda signals. The severity of the structural change is thus evaluated by its ability of deviating the direction of the acoustic energy, which is quantified with the scattering cross-section γ_e. A large change in the structure scatters more energy, hence results in modifications both in the early and late coda. On the other hand, if the structural change is small, it will scatter little energy and this will not cause significant modifications in the early coda, if any. However, the longer the wave propagates in the medium, the more it interacts with the change, thus accumulating more and more modifications in the coda as compared to the wave that propagates in the medium directly from the source to the receiver. Hence a small change will result in modifications in the late coda. This is illustrated in Fig. 1a, which shows typical time traces of coda waves in a multiple scattering medium in the presence and absence of a small structural change.

Such changes in the coda wave can be quantified by computing the decorrelation between signals monitored in the medium before and after the change. These will be denoted hereafter by $\phi_0(t)$ and $\phi_1(t)$, respectively. In practice, rather than computing the decorrelation between entire signals $\phi_0(t)$ and $\phi_1(t)$, these signals are first time-windowed into sub-signals of length ΔT, where ΔT is typically of the order of several mean free times. Estimating the waveform changes at different times in the coda corresponds to probing the medium at different depth, independently. The decorrelation between signals ϕ_0 and ϕ_1 over a time window of length ΔT centered at $t = t_j$ is defined such that:

$$DC^{exp}(t_j) = 1 - \frac{\int_{t_j-\Delta T/2}^{t_j+\Delta T/2} \phi_0(t)\phi_1(t)dt}{\sqrt{\int_{t_j-\Delta T/2}^{t_j+\Delta T/2} \phi_0(t)^2 dt \int_{t_j-\Delta T/2}^{t_j+\Delta T/2} \phi_1(t)^2 dt}}. \quad (1)$$

Fig. 1b shows the decorrelations between the signals represented in Fig. 1a for time windows centered at increasing instants t_j. Note that the value of decorrelations increases with the time in the coda, which is consistent with the above considerations regarding the depth penetration of the late coda, and the cumulative effect of the wave-change interaction.

Consider an experimental setup where N_k source-receiver pairs are used to measure the coda wave in a multiple scattering medium, and let N_j be the number of time windows used to compute the experimental decorrelations. Although current investigations indicate that there might exist an optimal value for the number of time windows N_j and their width ΔT, this is still an on-going research topic. Therefore, in the following these will be set heuristically to a few mean free times, as suggested in [14], in order to ensure a sufficient number of scattering events in the medium and inversion stability. Next, we assume that the experiment can be modeled faithfully. In that case if the decorrelations from the model fit the experimental ones, then the parameters in the model describe adequately the experimental parameters. These may include the geometrical properties of the changes themselves (positions, scattering cross-sections...), as well as the medium properties (diffusion constant, wave propagation speed...). Let us introduce $\mathbf{X}$, the variable that contains these parameters:

$$\mathbf{X} = \{p_1, p_2, \ldots, p_{N_i}\}^\dagger, \quad (2)$$

where $p_i \in P_i$, the search space associated with parameter i, N_i is the number of parameters, and superscript symbol $\dagger$ holds for vector or