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Sparse wavefield reconstruction and source detection using Compressed Sensing

Olivier Mesnil [a,*], Massimo Ruzzene [a,b]

[a] D. Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States
[b] G. W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States

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ABSTRACT

The paper presents a Compressed Sensing technique for the reconstruction of guided wavefields. Structural inspections based on the analysis of guided wavefields have proven to be effective at detecting and characterizing damage. However, wavefield detection is often a time consuming process, which limits its practicality. The proposed reconstruction technique estimates the location of sources and structural features interacting with the waves from a set of sparse measurements. Such features include damage, described as a scattering source. The wavefield is reconstructed by employing information on the dispersion properties of the medium under consideration. The procedure is illustrated through a one-dimensional analytical example, and subsequently applied to the reconstruction of an experimental wavefield in a composite panel with an artificial delamination. The results confirm the ability of the technique to identify the defect, while reconstructing the wavefield with good accuracy using a significantly reduced number of measurements.

1. Introduction

Lamb wave-based inspection continues to draw significant attention within the Non-Destructive Evaluation (NDE) and Structural Health Monitoring (SHM) communities [1]. A first group of techniques employs arrays of transducers mounted on the structure at a predetermined set of points [2–6]. These techniques generally provide a fast estimate of the location of defects such as holes, cracks, or delaminations, but often provide limited information on their shape, size and thickness-wise location. A second group of techniques relies on wavefield detection and analysis [7–10]. In this context, a wavefield denotes a series of images describing the time or frequency evolution of a propagating wave. Wavefield techniques provide a wealth of information that can effectively locate as well as quantify damage [7,11]. However, wavefield detection is a timely process due the common need for multiple averages at each location to mitigate potentially low signal-to-noise ratios, and the large number of measurements required to avoid spatial aliasing and resolve the desired information. Therefore, there is a recognized need to reduce acquisition time by reducing the number of acquisitions. This is the focus of recent papers where the use of dedicated equipment is explored to speed up acquisitions by increasing the number of measurements per unit of time. Examples include the use of a multipoint Laser Doppler Vibrometer (LDV) [12,13], and of a galvanometer mirror system in conjunction with a high amplitude single frequency piezoelectric transducer [14]. This paper contributes to this objective by exploring a process that reduces the number of required measurements, possibly below the limits imposed by Nyquist sampling theorem. The proposed process is based on the hypothesis that the time response recorded at one point of the surface of a plate structure can be employed to infer the response at any other point assuming prior knowledge of dispersion relations, plate geometry, and location of the excitation. When this knowledge is incomplete due to the presence of defects for example, this extrapolation is not straightforward and tools such as Compressed Sensing [15,16] must be used. Compressed Sensing (CS) is a mathematical theory commonly used as a reconstruction process for data sampled below the Nyquist frequency. Compressed Sensing deals with "sparse" or "compressible" signals randomly sampled within the domain of interest, either time or space. A "sparse" signal is here intended as a signal with only a few nonzero coefficients, whereas a signal is "compressible" if there exist a basis in which the signal has a sparse representation. Applications of CS include image processing [17] and magnetic resonance imaging [18]. Prior applications of CS to the reduction of wavefield measurements can be found in [19], where, in contrast to the present

* Corresponding author.
E-mail address: omesnil3@gatech.edu (O. Mesnil).

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paper, no prior knowledge of the physical properties of the medium such as its dispersion relations is assumed. A more recent paper from the same group [20] presents a wavefield reconstruction technique similar to the one presented herein, but using analytical dictionaries such as wavelets, Fourier functions or Gabor atoms as bases for wavefield reconstruction. Another series of papers develops a technique called sparse "wavenumber analysis" [21,22] for reconstructing the dispersion relations of a plate through sparse measurements. This technique has also been recently applied to pristine wavefield reconstruction [23]. The present paper presents a process to reconstruct a wavefield on an oversampled grid from few measurements. The paper differentiates itself from prior work by performing the reconstruction upon locating the non-pristine material points interacting with the wavefronts. The estimated dispersion relations of the medium are then used to extrapolate the wavefield onto a grid of points of arbitrary size and density. The estimated dispersion relations form the basis that sparsifies the wavefield, and allow wavefield reconstruction. Furthermore, this basis locates all features within the plate that interact or affect the wavefield. These include sources or scatterers such as damage. The number of measurements required for the reconstruction is significantly smaller than the number of measurements required by the common sampling requirements.

The paper is organized as follows. Following this introduction, Section 2 provides a summary of the CS formulation necessary for the development of the process. Section 3 contains the details of the matrix formulation and of the wavefield reconstruction process. Next, Section 4 illustrates the application of the reconstruction process to a one-dimensional (1D) analytically generated wavefield with a reflector, and presents results on a two-dimensional (2D) experimental wavefield on a composite panel containing an artificial delamination. Finally, Section 5 summarizes the main findings of the work and provides recommendation for future investigations.

2. Overview of Compressed Sensing

The fundamental mathematical result of Compressed Sensing (CS) states that if a signal $\boldsymbol{x} \in \mathbb{R}^{N\times 1}$ is K sparse, *i.e.* only K components of x are non zero, it can be exactly reconstructed with an overwhelming probability from few linear measurements randomly chosen [15,16]. The same result holds for compressible signals, which are signals well approximated by a small number of coefficients in a given basis. The basis in which a compressible signal is sparse is called a "sparsifying basis". In here and in the remainder of the paper, bold lower case letters denote vectors, while capitalized bold letters will be use for matrices.

The general under-sampling problem can be expressed as:

$$\boldsymbol{y} = \boldsymbol{\Phi x} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{R}^{M\times 1}$ is the measurement vector, $\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}$ is the down-sampling measurement matrix and $\boldsymbol{x} \in \mathbb{R}^{N\times 1}$ is the unknown vector to reconstruct. It is assumed that $M < N$, so that $\boldsymbol{\Phi}$ is a "short and large" type of matrix. In other words, the number of linear measurements is smaller than the number of unknown variables in $\boldsymbol{x}$.

For a compressible signal, the problem can be rewritten through a change of basis as follows:

$$\boldsymbol{y} = \boldsymbol{\Phi B \alpha} = \boldsymbol{A\alpha} \tag{2}$$

where $\boldsymbol{\alpha} \in \mathbb{R}^{P\times 1}$ is the sparse representation of $\boldsymbol{x}$ in the basis defined by $\boldsymbol{B} \in \mathbb{R}^{N\times P}$, *i.e.* $\boldsymbol{A} = \boldsymbol{\Phi B}$, where $\boldsymbol{A} \in \mathbb{R}^{M\times P}$ is the denoted as the "sensing matrix". Thus:

$$\boldsymbol{x} = \boldsymbol{B\alpha}$$

with P denoting the number of vectors forming the basis $\boldsymbol{B}$. Eq. (2) defines a CS problem. The objective is the estimation of the basis coefficients $\boldsymbol{\alpha}$ that provide the best reconstruction of the unknown set of physical variables $\boldsymbol{x}$ represented through $\boldsymbol{B}$. Inputs to the problem are the measurements $\boldsymbol{y}$. Several algorithms have been proposed for the solution of CS problems as in Eq. (2). For example, greedy algorithms such as the Orthogonal Matching Pursuit is proposed in [24,25]. As an alternative, $l1$-minimization algorithms such as Basis Pursuit have shown suitability for the reconstruction of noisy sparse signals [26,27]. Finally, Total Variation (TV) algorithms have been employed when the gradient of the signal to reconstruct is sparse [28,29]. An $l1$-minimizer is selected for the solution of the CS problem in this paper due to its robustness in the presence of measurement noise. The application of $l1$-minimizers requires that the sensing matrix $\boldsymbol{A}$ verifies the Restricted Isometry Property (RIP) with a RIP constant δ_s smaller than unity [30]. This constant is defined for a matrix $\boldsymbol{A}$ by the smallest scalar verifying the following inequality for all $\boldsymbol{y}$ and for all sub-matrices of $\boldsymbol{A}$, denoted $\boldsymbol{A}_s$:

$$(1-\delta_s)\|\boldsymbol{y}\|_2^2 \leqslant \|\boldsymbol{A}_s\boldsymbol{y}\|_2^2 \leqslant (1+\delta_s)\|\boldsymbol{y}\|_2^2$$

The δ_s constant is a characterization of the nearly orthogonal matrices operating on sparse vectors. The case $\delta_s \approx 0$ corresponds to a nearly orthonormal matrix, while $\delta_s \approx 1$ indicates that some of the vectors forming the matrix $\boldsymbol{A}$ are nearly identical or redundant. This requirement guarantees that the CS problem can be inverted with an overwhelming probability through $l1$-minimization. There exists a list of candidate matrices $\boldsymbol{A}$ obeying this property. These include the random Gaussian, the Bernoulli and the partial Fourier matrices [27]. However, computing these RIP constants is a non-deterministic polynomial-time hard problem and is not possible for most matrices [31]. In practice this requirement is often replaced by ensuring that matrices $\boldsymbol{B}$ and $\boldsymbol{\Phi}$ are incoherent [32,30]. This is mathematically verified by checking that the coherence, i.e. the maximum value of the scalar product between all the columns of the $\boldsymbol{A}$ matrix, is smaller than a constant defined in [33], meaning that the matrix $\boldsymbol{A}$ is nearly orthonormal.

The Basis Pursuit Denoising (BPDN) [26,27] algorithm is a $l1$-minimization algorithm that guarantees exact reconstruction [34] by solving the following problem

$$min\|\boldsymbol{\alpha}\|_1 \text{ subject to } \|\boldsymbol{y} - \boldsymbol{A\alpha}\|_2^2 \leqslant \sigma \tag{3}$$

where σ is a constant related to the noise level in the measurements, with $\sigma = 0$ in the absence of noise. The BPDN algorithm used is this paper is SGPL1, which is a solver for large scale sparse reconstruction that employs convex optimization to find a sparse representation of $\boldsymbol{\alpha}$ even when $\boldsymbol{B}$ is an over-complete dictionary of basis functions [35,36]. Accordingly, an estimation of the sparse optimum can be found even if the vectors in $\boldsymbol{B}$ are coherent with one another. As many of the BPDN solvers, SPGL1 is robust to measurement noise and takes σ as the only input parameter.

3. Guided wavefield reconstruction through Compressed Sensing

We formulate the reconstruction process by considering a wavefield as a compressible signal in space. Accordingly, let P be the number of regularly spaced pixels on which a wavefield is defined. The goal is to reconstruct the wavefield from the knowledge of M sparse measurements such that $M < P$. The dispersion relations of the media and the Lamb wave propagation equation are used to formulate a sparsifying basis for the wavefield. In the proposed process, location of the sources are first estimated from M measurements by means of a $l1$-minimizer and the sparsifying basis (Eq. (3)). The estimated sources are then used to extrapolate, or reconstruct, the wavefield over a desired grid of P points, which