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Wave propagation in piezoelectric layered structures of film bulk acoustic resonators

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ABSTRACT

In this paper, we studied the wave propagation in a piezoelectric layered plate consisting of a piezoelectric thin film on an electroded elastic substrate with or without a driving electrode. Both plane-strain and anti-plane waves were taken into account for the sake of completeness. Numerical results on dispersion relations, cut-off frequencies and vibration distributions of selected modes were given. The effects of mass ratio of driving electrode layer to film layer on the dispersion curve patterns and cut-off frequencies of the plane-strain waves were discussed in detail. Results show that the mass ratio does not change the trend of dispersion curves but larger mass ratio lowers corresponding frequency at a fixed wave number and may extend the frequency range for energy trapping. Those results are of fundamental importance and can be used as a reference to develop effective two-dimensional plate equations for structural analysis and design of film bulk acoustic resonators.

1. Introduction

Acoustic wave resonators made from piezoelectric crystals are key components of circuits for alternating currents and have been widely used for time keeping, frequency operation, signal generation and processing as frequency standards. During the last one to two decades, researchers succeeded in depositing with good enough quality a thin piezoelectric film of AlN or ZnO with proper electrode configuration on a silicon layer to form thin film bulk acoustic wave resonators (FBARs) [1–3]. Compared with conventional acoustic wave resonators, FBARs have a series of advantages such as much smaller size and compatible fabrication with other on-chip and integrated circuit (IC) technologies and have become research hot recently. There are several structural types of FBARs operating with thickness modes [4–8] or solidly mounted on an elastic substrate [9,10]. Fig. 1 shows the structure of a typical, basic, and widely used type of FBARs which is the one that is going to be studied in the current paper. Structurally, the FBAR in Fig. 1 is a multilayered plate with metal electrodes, a piezoelectric film, and an elastic layer. The *c*-axis of the piezoelectric film is in the x_3 direction. To aid the analysis and design of FBARs, it is necessary to obtain accurate predictions of their frequency and mode shapes. Starting from the three-dimensional linear piezoelectric equations, however, direct solution to such a multilayered structure is still mathematically changing, no matter theoretically or numerically [11–13]. Up to now, most theoretical analyses on FBARs are based on one-dimensional models which can describe the most basic vibration characteristics of FBARs [7,14–18]. However, for real devices of finite plates, one-dimensional models are inadequate. They cannot describe the in-plane mode variations associated with finite plates. They are also incapable of describing mode couplings induced by wave reflections at the plate edges in finite devices. Recently, Qian's group [19] has studied the free vibrations of FBAR multilayered structures by using the Steven–Tiersten's two-dimensional scalar differential equations [20] which are mathematically simple and accurate, and can describe the in-plane variation of the operating thickness-extensional mode and the related energy trapping in FBARs. The main limitation of the scalar equation is that it is for the single operating mode of the FBAR only and therefore cannot describe mode coupling, although it has in-plane mode variation in the model [20]. At present, the design and operation of FBARs are adversely affected by various couplings between the operating mode and other unwanted modes. The understanding of the mechanism of these mode couplings is limited. Therefore, it is extremely needed to develop effective two-dimensional plate equations which can simultaneously describe both the in-plane mode couplings and the energy trapping vibration phenomenon. One way is to follow Mindlin's approach in treating mode couplings and in-plane mode variations in conventional quartz resonators [21–25]. The structures of FBARs are multilayered and hence are more complicated than single-layered

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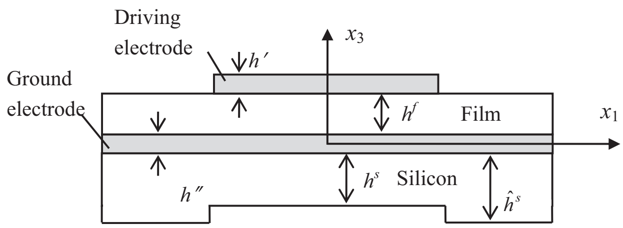

Fig. 1. Cross section of a typical thin AlN or ZnO film on a silicon layer as an FBAR.

quartz resonators, for which we first need to obtain accurate results of dispersion curves and cut-off frequencies in the wave number range of interest for the operation of FBARs. This motivates the current work in this paper.

Since the structures of FBARs shown in Fig. 1 have two different regions, with or without a top driving electrode, we consider wave propagation in plate models for FBARs with a top electrode layer and without a top electrode layer, respectively. Both plane and anti-plane waves were taken into account. From the three-dimensional piezoelectro-elastic equations, an exact procedure was established to calculate dispersion relations, cut-off frequencies and vibration distributions of selected modes. Those results are of fundamental importance and can be used as a reference to improve the accuracy of the two-dimensional plate equations by requiring the cutoff frequencies and the curvatures of the dispersion curves at cutoff frequencies predicted by the two-dimensional plate equations and the three-dimensional equations to be the same.

2. Theoretical derivation

Consider the composite plates in Fig. 2. We study straight-crested waves without x_2 dependence, i.e., $\partial/\partial x_2 = 0$. In this case, the equations of linear piezoelectricity for ZnO or other crystals of class 6 mm with the c axis along x_3 decouples into two groups. One gives the displacement components u_1 and u_3 as well as the electric potential φ, which corresponds to plane-strain wave case. The other is for u_2 alone which corresponds to anti-plane wave case. The two cases will be dealt with below, respectively.

2.1. Plane-strain waves

The relevant equations of motion and the charge equation of electrostatics are

$$\begin{aligned} &T_{11,1} + T_{31,3} = \rho \ddot{u}_1, \\ &T_{13,1} + T_{33,3} = \rho \ddot{u}_3, \\ &D_{1,1} + D_{3,3} = 0, \end{aligned} \tag{1}$$

where the stress components T_{ij} and the electric displacement components D_i are related to the displacement and electric displacement gradients through the constitutive relations

$$\begin{aligned} &T_{11} = c_{11}u_{1,1} + c_{13}u_{3,3} + e_{31}\varphi_{,3}, \\ &T_{33} = c_{13}u_{1,1} + c_{33}u_{3,3} + e_{33}\varphi_{,3}, \\ &T_{31} = T_{13} = c_{44}(u_{3,1} + u_{1,3}) + e_{15}\varphi_{,1}, \end{aligned} \tag{2}$$

and

$$\begin{aligned} &D_1 = e_{15}(u_{3,1} + u_{1,3}) - \varepsilon_{11}\varphi_{,1}, \\ &D_3 = e_{31}u_{1,1} + e_{33}u_{3,3} - \varepsilon_{33}\varphi_{,3}. \end{aligned} \tag{3}$$

The substitution of Eqs. (2) and (3) into Eq. (1) gives

$$\begin{aligned} &c_{11}u_{1,11} + c_{44}u_{1,33} + (c_{13} + c_{44})u_{3,13} + (e_{31} + e_{15})\varphi_{,13} = \rho \ddot{u}_1, \\ &c_{44}u_{3,11} + c_{33}u_{3,33} + (c_{44} + c_{13})u_{1,31} + e_{15}\varphi_{,11} + e_{33}\varphi_{,33} = \rho \ddot{u}_3, \\ &(e_{15} + e_{31})u_{1,13} + e_{15}u_{3,11} + e_{33}u_{3,33} - \varepsilon_{11}\varphi_{,11} - \varepsilon_{33}\varphi_{,33} = 0. \end{aligned} \tag{4}$$

Similarly, for silicon layer which is a cubic crystal, from the equations of anisotropic elasticity we have

$$\begin{aligned} &c^s_{11}u_{1,11} + c^s_{44}u_{1,33} + (c^s_{13} + c^s_{44})u_{3,13} = \rho^s \ddot{u}_1, \\ &c^s_{44}u_{3,11} + c^s_{33}u_{3,33} + (c^s_{44} + c^s_{13})u_{1,31} = \rho^s \ddot{u}_3, \end{aligned} \tag{5}$$

where we have used a superscript 's' to indicate the material constants of silicon layer.

At the top of the composite plate where $x_3 = h^f$, mechanically the surface is free. Electrically the surface may be electroded and grounded, or unelectroded. For an electroded surface, the boundary conditions are

$$\begin{aligned} &-T_{31}(h^f) = \rho' h' \ddot{u}_1(h^f), \\ &-T_{33}(h^f) = \rho' h' \ddot{u}_3(h^f), \\ &\varphi(h^f) = 0, \end{aligned} \tag{6}$$

where ρ' and h' are the mass density and thickness of the top electrode. In Eq. (6), we have assumed that the electrodes are very thin and neglected their stiffness. For an unelectroded top surface, we have

$$T_{31}(h^f) = 0, \quad T_{33}(h^f) = 0, \quad D_3(h^f) = 0. \tag{7}$$

At the interface between the ZnO film and the silicon layer where $x_3 = 0$, the continuity conditions are

$$\begin{aligned} &u_1(0^+) = u_1(0^-), \quad u_3(0^+) = u_3(0^-), \quad \varphi(0^+) = 0, \\ &T_{31}(0^+) - T_{31}(0^-) = \rho'' h'' \ddot{u}_1(0), \\ &T_{33}(0^+) - T_{33}(0^-) = \rho'' h'' \ddot{u}_3(0), \end{aligned} \tag{8}$$

where ρ'' and h'' are the mass density and thickness of the interface electrode. At the bottom of the composite plate where $x_3 = -h^s$ the traction-free boundary conditions are

$$T_{31}(-h^s) = 0, \quad T_{33}(-h^s) = 0. \tag{9}$$

We look for time-harmonic wave solutions that may exist in the composite plate. The ZnO film and the silicon layer need to be treated separately initially, and then boundary and continuity conditions will be applied.

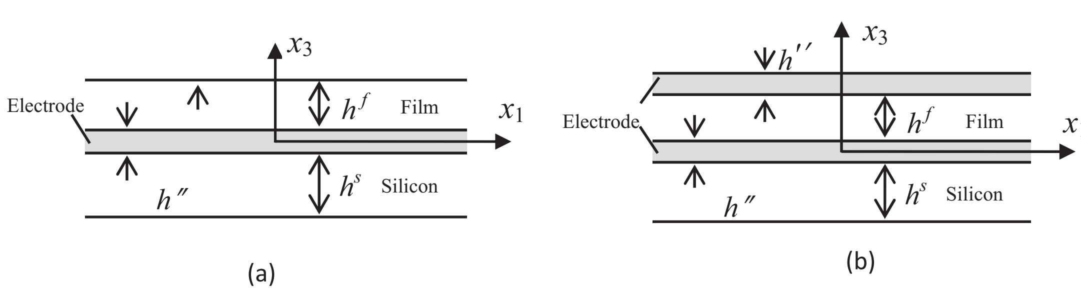

Fig. 2. Plate models for FBARs: (a) without a driving electrode; (b) with a driving electrode.