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A computer simulation study of soft tissue characterization using low-frequency ultrasonic tomography

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ABSTRACT

We investigate the potential of using ultrasonic diffraction tomography technique for characterization of biological tissues. Unlike most of other studies where ultrasonic tomography operates at frequencies higher than 1 MHz, low-frequency tomography uses lower frequencies on the order of 0.3–0.5 MHz. Such a choice is due to low attenuation at these frequencies, resulting in higher precision of input data. In this paper we explore transmission and reflection schemes for both 2D (layer-by-layer) and 3D tomography. We treat inverse tomography problems as coefficient inverse problems for the wave equation. The time-domain algorithms employed for solving the inverse problem of low-frequency tomography focus on the use of GPU clusters. The results obtained show that a spatial resolution of about 2–3 mm can be achieved when operating at the wavelength of about 5 mm even using a stationary 3D scheme with a few fixed sources and no rotating elements. The study primarily focuses on determining the performance limits of ultrasonic tomography devices currently designed for breast cancer diagnosis.

1. Introduction

The differential diagnosis of breast cancer is a problem of prime importance in modern medicine. Common medical devices for ultrasonic examination usually employ a reflection-based scheme [1–4]. In the simplest case, where sounding is performed with the emitter at a fixed position, the doctor sees an image in the form of a time sweep of the ultrasonic signal reflected from internal organs within a narrow angle. A transducer array is usually employed to this end. It seems a natural solution to collect the reflected signals by moving the transducer array around the object studied. One can use the resulting data to try to reconstruct the internal structure of the object studied. However, high-quality tomographic images cannot be reconstructed using reflection data alone. The analysis of various tomography schemes and their optimisation is a problem of vital importance whose solution was addressed in many publications [5–7].

Most of the published studies in this field are dedicated to the development of ultrasonic tomography devices operating at the frequencies of 1–3 MHz or higher. The use of ray-based models appears to be a reasonable approach in this frequency domain. Theoretical frameworks have been developed and prototypes have been made that implement these approaches in 2D layer-by-layer schemes [8–10] and in fully 3D schemes [11,12]. Inverse problems of ultrasonic tomographic diagnostics are nonlinear in contrast to those arising in X-ray tomography. Refraction puts into question the applicability of layer-by-layer models [13]. Attempts to solve inverse problems in the 3D formulation appear to offer better prospects. Three-dimensional inverse problems of ultrasonic tomography, which are much more computationally expensive, can be tackled with computing clusters equipped with graphics processing units (GPU) [14–16]. Attempts have been made to improve the results obtained using ray models [17]. The disadvantage of ray-based models is that they are incapable of describing such wave phenomena as diffraction and multiple scattering. Pratt et al. [18] attempted to estimate the impact of wave effects on the tomographic image reconstruction. Kretzek and Ruiteret al. [15] also try to solve ultrasonic tomography problems in the ray optics approximation. These authors use the velocity structure reconstructed in the ray approximation for reflectivity image reconstruction.

The use of models based on wave equations in ultrasonic tomography is a more promising approach. Inverse problems of ultrasonic tomography in wave-based models are nonlinear and many authors use various linearised approximations to solve them. The most common approach involves the use of the so-called Born approximation of the wave equation [19,20]. From the practical point of view, linearised approximations have a rather limited

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potential for solving nonlinear problems and can be used only in the neighborhood of the required solution, which is unknown in the case of inverse problems. The study [21] analyses the limiting capabilities of linearised Born-type models. However, from the theoretical point of view, linearised approximations can be used for qualitative analysis of tomography schemes [6,22].

At present, prototype ultrasonic tomography devices have been developed for the use in breast examinations [23–25] with the data interpreted in terms of nonlinear wave models. Wiskin et al. [23] use narrow-band ultrasonic wave sources with frequencies above 1 MHz. Approximate algorithms have been developed for solving the inverse problems of 3D ultrasonic tomography in the wave approximation of the Helmholtz equation. These algorithms have been tested on a prototype tomographic device for breast cancer diagnosis. Both transmission and reflection data are used to solve the inverse problem. The algorithms are implemented as a two-stage procedure. In the first stage, the spatial distribution of the speed of sound is reconstructed approximately using a transmission tomography scheme within the framework of a parabolic model in the form of the Helmholtz equation approximation for small refraction angles. In the second stage, an attempt is made to use the reflected signal to improve the derived approximations. Unlike Wiskin et al. [23], in this paper we demonstrate the feasibility of a low-frequency ultrasonic tomography device operating in the 300–500 kHz frequency range. We develop algorithms for solving three-dimensional nonlinear inverse problem in terms of the model of hyperbolic wave equation with no simplifying assumptions.

The authors of some studies [24] attempt to reconstruct both the velocity structure and the density distribution. It is shown that the velocity structure can be recovered better than the density distribution. In real objects attenuation is always present. We showed [25] that in the case of low attenuation in a model incorporating both diffraction and attenuation effects the velocity structure can be reconstructed better than the attenuating properties. In the case of low attenuation the reconstructed velocity structure depends only slightly on the attenuation model employed. The velocity structure can be reconstructed fairly well even if the input data errors are of about 5%, whereas the distribution of attenuating properties in the medium cannot be recovered in this case. Similar results follow from the studies of Wiskin et al. [26].

The aim of this study is to develop efficient algorithms for solving 2D and 3D problems of acoustic tomography in terms of wave models. Inverse problems are considered as coefficient inverse problems of the reconstruction of the velocity structure in the diagnosed region. From the mathematical point of view there are two approaches to solving inverse problems of ultrasonic tomography. On the one hand, one can try to develop algorithms using finite-difference schemes for differential equations. An alternative approach is represented by the well-known Green function method, which allows reformulating the inverse problem as a nonlinear operator equation [7,27]. The inverse problem of ultrasonic tomography in the Green function representation has a number of advantages including the possibility of a simple formulation of the problem as a set of nonlinear operator equations. The undoubted advantage of such a formulation is that it requires no boundary conditions to be imposed except for the natural Sommerfeld radiation condition at infinity.

However, the integral approach has a very important disadvantage due to the extremely computationally expensive nature of the algorithms involved. Lavarello and Oelze [7] showed that the number of operations in the iterative algorithm proposed for solving the nonlinear problem scales as $O(N^6)$ in the 3D case and $O(N^5)$ in the 2D case, where N is the number of grid points along one dimension. We obtained a similar result in our study [27], where we used an iterative process based on the Newton method. The strong dependence of the number of operations on the number of grid points forced us to solve inverse problems on a $50 \times 50 \times 50$ grid. In real-life applications a $400 \times 400 \times 400$ or denser grid is needed to address the 3D problem of ultrasonic tomography, resulting in the increase of the computational time by a factor of several tens of thousands. The situation cannot be remedied even by using a supercomputer. Even with a factor of 1000 speedup compared to a PC, a supercomputer would allow the number of grid points to be increased by a factor of $\sqrt[6]{1000} \approx 3$ along each dimension. This is a typical problem for the integral approach.

It follows from the above that the development of algorithms that can be run within practically feasible time on modern computers is of great importance in three-dimensional inverse problems of ultrasonic tomography. The breakthrough results in the solution of inverse problems of wave tomography are associated with the studies that demonstrate the possibility of exact computation of the gradient of the residual functional by solving a 'conjugate' problem [19,28,29]. In this paper we use gradient-based minimisation methods to solve the nonlinear problem as a coefficient inverse problem for differential equation. The algorithms employed are based on finite-difference time-domain (FDTD) numerical methods and require only $O(N^4)$ operations to solve the three-dimensional problem.

2. Formulation of the direct and inverse problems of ultrasonic tomography and numerical algorithms employed in the two- and three-dimensional case

Fig. 1 illustrates the arrangement of the sources and detectors for the three-dimensional inverse problem. Number 1 denotes the sources and number 2 denotes the detectors of ultrasonic radiation, which are located on the faces of the cube Ω. We assume that the object G is located inside the cube Ω. The remaining space L is filled with water with known sound speed v_0. Fig. 2 illustrates the arrangement of the sources and detectors for the layer-by-layer tomography scheme, where the three-dimensional problem is replaced by a set of two-dimensional problems. Numbers 1 and 2 in Fig. 2 denote the sources and detectors, respectively; G is the domain under study, and L is the domain with known sound speed v_0.

In this study, we address the inverse problem using the wave approximation in the time-domain formulation with point sources. Acoustic pressure field $u(\boldsymbol{r},t)$ in the domain $\Omega \subset \mathbb{R}^N$ $(N = 2,\ 3)$ produced by a point source located at point $\boldsymbol{r}_0$ and generating a pulse described by function $f(t)$, obeys the wave equation:

$$c(\boldsymbol{r})u_{tt}(\boldsymbol{r},t) - \Delta u(\boldsymbol{r},t) = \delta(\boldsymbol{r} - \boldsymbol{r}_0) \cdot f(t), \tag{1}$$

$$u(\boldsymbol{r},t=0) = u_t(\boldsymbol{r},t=0) = 0, \tag{2}$$

$$\partial_n u|_{\Gamma T} = p(\boldsymbol{r},t), \tag{3}$$

where t is the time, $0 < t < T$; u is the acoustic pressure; $c(\boldsymbol{r}) = v^{-2}(\boldsymbol{r})$, $v(\boldsymbol{r})$ is the sound speed in the medium; Δ is the Laplace operator with respect to $\boldsymbol{r} \in \mathbb{R}^N$ $(N = 2, 3)$, Γ is the boundary of the domain N; $\partial_n u|_{\Gamma T}$ is the derivative along the normal to the surface Γ in the region $\Gamma \times (0,T)$, and $p(\boldsymbol{r},t)$ is a known function.

The two-dimensional wave equation (for $N = 2$) describes a three-dimensional problem that is independent of one of the coordinates. For example, if the object does not vary along one of the dimensions and is sounded with cylindrical waves. The two-dimensional version of the model can be used as an approximation for 3D computations if the object under study varies only slightly along one of the dimensions. In medical tomography, where the structure and parameters of an irregularity have to be