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Guided wave mode selection for inhomogeneous elastic waveguides using frequency domain finite element approach

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ABSTRACT

This article describes the use of the frequency domain finite element (FDFE) technique for guided wave mode selection in inhomogeneous waveguides. Problems with Rayleigh–Lamb and Shear-Horizontal mode excitation in isotropic homogeneous plates are first studied to demonstrate the application of the approach. Then, two specific cases of inhomogeneous waveguides are studied using FDFE. Finally, an example of guided wave mode selection for inspecting disbonds in composites is presented. Identification of sensitive and insensitive modes for defect inspection is demonstrated. As the discretization parameters affect the accuracy of the results obtained from FDFE, effect of spatial discretization and the length of the domain used for the spatial fast Fourier transform are studied. Some recommendations with regard to the choice of the above parameters are provided.

1. Introduction

Ultrasonic guided waves are widely used as a tool for a variety of problems in nondestructive evaluation (NDE) and structural health monitoring (SHM). As opposed to bulk waves which travel in unbounded media, guided waves require boundaries to guide the energy in the structure. Also, there exists multiple guided wave modes in the structure owing to the nature of the eigenvalue problem associated with the structure. Hence, guided wave mode-selection [1] is the key to identify specific kinds of defects. For this, it is necessary to have a thorough understanding of the characteristics of wave propagation in waveguides. Two essential features that characterize the wave propagation in a homogeneous waveguide, where there are no changes (material or geometric) along the length of the waveguide, are dispersion curves and wavestructures [2].

While dispersion curves depict the frequency-wavenumber (ω, k) combinations at which propagating guided wave modes exist in the structure, wavestructures describe the through-thickness displacement/stress profiles for the guided wave modes. Both dispersion curves and wavestructures guide the choice of a mode and frequency for a particular application. While analytical solutions to obtain dispersion curves and wavestructures are available for simple geometries like those of homogeneous plates and pipes [2,3], numerical methods are the only resort for more complicated geometries. Several approaches have been used in the past to address the above issue [4]. The finite difference approach and the finite element method (FEM) [5] are by far the most commonly used numerical techniques to study wave propagation in inhomogeneous waveguides. Even though time-domain transient dynamic analysis using FEM has the capability to simulate wave propagation in structures of large sizes and complex shapes, it requires a lot of computational effort in most cases. Hence, other approaches such as the semi-analytical finite element method (SAFE) [6], which is a hybrid approach combining analytical and finite element methods, are being used to study homogeneous waveguides. While it is possible to obtain dispersion curves and wavestructures for waveguides which are homogeneous in the propagation direction using SAFE, inhomogeneous waveguides, where the material or geometry changes along the length of the waveguide, call for a different approach. Of particular interest are functionally graded materials, inhomogeneous periodic waveguides and waveguide transitions [7].

For example, the problem of guided wave mode conversion across a discrete waveguide transition [7] is studied using SAFE and the normal mode expansion technique. The approach was to use solutions from SAFE to model the wave field in the homogeneous regions before and after the transition and then use the appropriate boundary conditions at the transition to determine the modal content in the respective wave fields. It is noted that such a transition is a rather simple inhomogeneity and such good workable solutions are not known for more complex inhomogeneous waveguides, impairing the efficiency of NDE/SHM in

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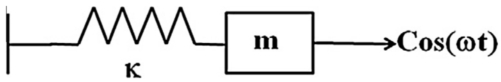

Fig. 1. Schematic of spring-mass system.

inspecting them. This article demonstrates the use of a frequency domain finite element (FDFE) approach for guided wave mode selection for NDE/SHM of inhomogeneous waveguides. The FDFE approach has previously been used to study the scattering from defects and reflection and transmission of waves across interfaces [8–10] and waveguide transitions [11]. The corresponding results are compared with those from time-domain finite element approach and are found to be in good agreement. To the best of our knowledge, the above approach has not been employed for guided wave ultrasonic inspection, so we address this practical issue here. The advantages of FDFE relative to time-domain analysis include: computational efficiency, it permits study of mode conversion, it gives phase velocities as opposed to group velocity, and it enables an efficient method to select modes sensitive to damage.

The content of this article is organized as follows. Section 2 presents the background, theory and formulation of the elastic boundary value problem in the frequency domain. Then Section 3 demonstrates the approach for an isotropic homogeneous plate where the phase velocities and wavestructures of Rayleigh–Lamb (RL) and Shear-Horizontal modes (SH) are recovered. Next, Section 4 presents the results obtained from applying FDFE for two cases of inhomogeneous waveguides. There are many types of inhomogeneous waveguides that are of practical importance for NDE and SHM. Damage localization is one good example. The change in energy transmitted through the damaged region is necessary for through-transmission methods and the wave energy reflected by the damaged region is required for pulse-echo methods. The idea of mode selection using FDFE is outlined and the special case of a disbond in an adhesively bonded joint between composite laminates is presented as an example. The effect of parameters such as discretization on the solution of FDFE is presented in Section 5 and the conclusions are drawn in Section 6.

2. Frequency domain finite element approach (FDFE)

The governing equations for a linear elastic waveguide [12] are given by

$$\nabla \cdot \sigma = \rho \ddot{u} \quad \text{(Balance of linear momentum)} \tag{1}$$

$$\sigma = C\epsilon \quad \text{(Constitutive relation)} \tag{2}$$

$$\epsilon = \frac{1}{2}(\nabla u + (\nabla u)^T) \quad \text{(Strain-displacement relation)} \tag{3}$$

where σ denotes Cauchy stress, C represents the tensor of elastic moduli, ϵ is the strain tensor, u denotes the displacement field and ρ denotes the mass-density of the material. The frequency domain finite element approach attempts to solve Eqs. (1)–(3) in the frequency domain. Solution in the frequency domain gives the characteristics of a system in terms of the frequency response function/modal response function. Just as one can determine a frequency response function for a particle system, one could extend a similar concept to characterize the structural behavior of the waveguide.

Consider the forced vibration of a particle attached to a spring as shown in Fig. 1. The frequency response function can be written as $H(\omega) = 1/(\kappa - m\omega^2)$, where κ, m, and ω are stiffness, mass, and frequency respectively. The natural frequency of the system is obtained by setting $\kappa - m\omega^2 = 0$. Likewise, a continuum can be thought of as an infinite system of particles and hence it has infinite natural frequencies. Similarly, a waveguide could be thought of as a vibrating continuum whose response can be captured in a modal response function/transfer function, which could be described as $H(\omega, k)$ where ω denotes the frequency and k the wavenumber. The propagating wave modes can be identified as zeros of the transfer function $H(\omega, k)$. For example, $H(\omega, k)$ for the wave propagation in an isotropic, homogeneous, traction free plate [2] is given by $H(\omega, k) = (D_s(\omega, k)D_a(\omega, k)D_{SH}(\omega, k))^{-1}$, where

$$D_s(\omega, k) = \frac{tan(qh)}{tan(ph)} + \frac{4k^2pq}{(q^2 - k^2)^2} \quad \text{Symmetric RL modes} \tag{4}$$

$$D_a(\omega, k) = \frac{tan(qh)}{tan(ph)} + \frac{(q^2 - k^2)^2}{4k^2pq} \quad \text{Antisymmetric RL modes} \tag{5}$$

$$D_{SH}(\omega, k) = qh - \frac{n\pi}{2}; \; n = 1, 2, \cdots; \quad \text{SH modes} \tag{6}$$

where $p = \sqrt{\left(\frac{\omega^2}{c_l^2} - k^2\right)}$ and $q = \sqrt{\left(\frac{\omega^2}{c_t^2} - k^2\right)}$, with c_l and c_t representing the longitudinal and transverse wave speeds in the material.

The function $H(\omega, k)$ could be explicitly determined either analytically or numerically as above in very few cases. For the case of inhomogeneous waveguides or waveguides with complex geometries, it may not be possible for a single wavenumber k to characterize the displacement field in the entire waveguide at a given frequency ω owing to the inhomogeneous nature of the waveguide. Hence it is an onerous task to identify a particular mode suitable for a given NDE/SHM application. One alternative approach is to study the wave propagation for various excitations (boundary conditions) using time-domain FEM techniques, use the time-domain data to determine the displacement field, and then apply a 2DFFT (spatio-temporal fast Fourier transform) to determine the frequency-wavenumber content in the displacement field.

The other approach, i.e., FDFE, attempts to solve the problem for time harmonic excitations; i.e., in the frequency domain. It assumes a time harmonic nature of displacement, stress and strain fields and solves Eqs. (1)–(3) at each frequency. To be specific, we assume $u = u(\hat{x})e^{i\omega t}$ and $\sigma = \sigma(\hat{x})e^{i\omega t}$ where $\hat{x}$ denotes the position of the material point in Cartesian space (x, y, z). The balance of linear momentum under the above assumptions simply becomes

$$\nabla.\sigma(\hat{x}) + \rho\omega^2 u(\hat{x}) = 0 \tag{7}$$

Note that time-derivatives are eliminated with the time-harmonic assumption. Importantly for the problems at hand, any inhomogeneities in the material properties or the geometry can be incorporated in the finite element discretization. Eq. (7) is solved using FEM to obtain $u(\hat{x})$ at each frequency ω. The advantage of this approach is that the computational cost is considerably reduced because we end up solving a pseudo-static problem that requires no time-stepping as in the time-domain finite element

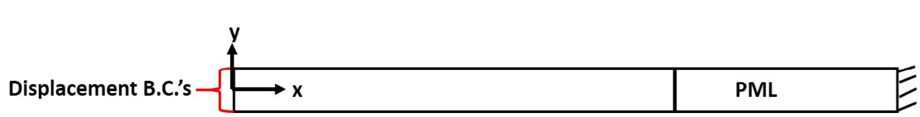

Fig. 2. Schematic of the model used for the simulations.