Ultrasonics 63 (2015) 47–53

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Effect of initial stress on propagation behaviors of shear horizontal waves in piezoelectric/piezomagnetic layered cylinders

X. Zhao [a,b], Z.H. Qian [a,*], S. Zhang [b], J.X. Liu [b]

[a] *State Key Laboratory of Mechanics and Control of Mechanical Structures/College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China*

[b] *Department of Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, PR China*

ARTICLE INFO

Article history:
Received 13 June 2015
Accepted 15 June 2015
Available online 23 June 2015

Keywords:
SH wave
Piezoelectric
Piezomagnetic
Layered cylinder
Initial stress

ABSTRACT

An analytical approach is taken to investigate shear horizontal wave (SH wave) propagation in layered cylinder with initial stress, where a piezomagnetic (PM) material thin layer is bonded to a piezoelectric (PE) cylinder. Two different material combinations are taken into account, and the phase velocities of the SH waves are numerically calculated for the magnetically open and short cases, respectively. It is found that the initial stress, the thickness ratio and the material performance have a great influence on the phase velocity. The results obtained in this paper can offer fundamental significance to the application of PE/PM composite media or structure for the acoustic wave and microwave technologies.

1. Introduction

PE/PM composites (also called magneto-electro-elastic (MEE) materials) are composed of PE and PM materials in some way, which not only have the properties of PE and PM materials, but also the abilities to convert the energies between electric and magnetic fields, namely, possess the magneto-electric (ME) effect. Since van Suchtelen [1], van den Boomgaard et al. [2] and van den Boomgaard et al. [3] investigated the ME effect of the PE/PM composites, many researchers have devoted to predict and determine the ME effect theoretically and experimentally. Nan et al. [4] and Priya et al. [5] have taken comprehensive investigations on this topic.

Because of the ME coupling effect, PE/PM or MEE composites can be widely applied in sensors, actuators, acoustic devices, and transducers, and so on. Usually, those devices work based on elastic wave propagations [6,7]. Therefore, it is of great importance to analyze the propagation behaviors of elastic waves in MEE composite structures, which attracted a lot of investigations. These studies include the surface wave in MEE composite structure [8–10], the interfacial wave in MEE bi-materials with perfect or imperfect interface [11–15], the guided wave in multilayered structure [16,17]. Similar studies when considering the inhomogeneous properties of MEE materials have also been reported in [18,19].

In the above-mentioned studies, the composites or structures are usually considered to be flat layered. Surface acoustic wave devices of such flat layered structures usually have the desired time delay limited by the length of the crystal. However, if the surface wave is guided around a cylindrical body, one will be able to achieve a much longer time delay. Therefore, the analysis of the wave propagation in cylindrical structures has become very important. Du et al. [20] and Sun et al. [21] successively investigated the propagation behaviors of SH waves in MEE layered cylinders.

Furthermore, due to the non-uniform material properties and unmatched coefficients of thermal expansion, there inevitably exists residual stresses during the manufacture process of acoustic surface wave devices. On the other hand, to prevent the composites from brittle fracture, the layered structures are usually pre-stressed during the manufacture process. It is well known that initial stress in the layered structures will lead to microcrack et al. and affect the propagation characteristics of elastic waves. To study the effect of initial stresses on the propagation of elastic waves, two methods are often used. One is the incremental field theory which is originally derived by Tiersten [22] and Nelson [23], in which constitutive relations of materials are changed by initial stresses. Whilst, in the incremental field theory the third-order or even higher-order nonlinear material constants are needed for a complete description of the lowest order effect of biasing fields [24]. But these constants are usually difficult to obtain and only known for a few materials currently [24]. Therefore, many researchers prefer the other often-used method,

* Corresponding author. Tel.: +86 25 84892696.
E-mail addresses: qianzh@nuaa.edu.cn, zhenghua_qian@163.com (Z.H. Qian).

http://dx.doi.org/10.1016/j.ultras.2015.06.012
0041-624X/

i.e., the initial stress theory [25] in which initial stresses are included in equilibrium equations and do not change constitutive relations of materials. Using the initial stress theory, Liu et al. [26] investigated the propagation behavior of Love wave in a layered PE structure with an initial stress, Du et al. [27] analyzed the propagation of Love waves in prestressed piezoelectric layered structures loaded with viscous liquid, Guo and Wei [28] studied the effects of initial stress on the reflection and transmission waves at the interface between two PE half-spaces. For MEE or PE/PM composite structures, Du et al. [29] studied the propagation of Love wave in layered PE/PM structures with constant initial stress, and showed the effect of the initial stress on the Love wave propagation. For the similar structure, Zhang et al. [30] solved the coupled MEE field equations by adopting the Wentzel–Kramers–Brillouin (WKB) method, and investigated the effect of inhomogeneous initial stress on Love wave propagation.

In the present work, we will analytically investigate the propagation behaviors of SH waves in a cylindrical PE/PM composite with initial stresses by the initial stress theory. The effects of the initial stresses, material properties and electromagnetic boundary conditions on the dispersion relations are discussed in detail. The results obtained in this paper will be useful for both theoretical research and engineering application of SH waves.

2. Statement of the problem

Here, a cylindrical PE/PM composite is taken into account, as shown in Fig. 1. It consists of a PE cylinder and a concentric PM covering layer. The inner and outer radii of the cylindrical structure are a and b, respectively. The PE and PM materials are both polarized along the x_3-axis, perpendicular to the x_1–x_2 plane. In the later derivation, the cylindrical coordinates r, θ, z will be used.

2.1. Basic equations

Consider the SH waves propagating along the θ direction, where the anti-plane elastic field is coupled with the in-plane electromagnetic field. Hence, the constitutive equations can be expressed as follows [21]

$$\begin{aligned}&\sigma^e_{rz}=G_e\frac{\partial u^e}{\partial r}+e_{15}\frac{\partial\varphi^e}{\partial r},\quad \sigma^e_{\theta z}=\frac{1}{r}\left(G_e\frac{\partial u^e}{\partial\theta}+e_{15}\frac{\partial\varphi^e}{\partial\theta}\right),\\&D^e_r=e_{15}\frac{\partial u^e}{\partial r}-\kappa_e\frac{\partial\varphi^e}{\partial r},\quad D^e_\theta=\frac{1}{r}\left(e_{15}\frac{\partial u^e}{\partial\theta}-\kappa_e\frac{\partial\varphi^e}{\partial\theta}\right),\\&B^e_r=-\mu_e\frac{\partial\phi^e}{\partial r},\quad B^e_\theta=-\frac{1}{r}\mu_e\frac{\partial\phi^e}{\partial\theta},\end{aligned}\tag{1}$$

for the PE material, and

$$\begin{aligned}&\sigma^m_{rz}=G_m\frac{\partial u^m}{\partial r}+h_{15}\frac{\partial\phi^m}{\partial r},\quad \sigma^m_{\theta z}=\frac{1}{r}\left(G_m\frac{\partial u^m}{\partial\theta}+h_{15}\frac{\partial\phi^m}{\partial\theta}\right),\\&D^m_r=-\kappa_m\frac{\partial\varphi^m}{\partial r},\quad D^m_\theta=-\frac{1}{r}\kappa_m\frac{\partial\varphi^m}{\partial\theta},\\&B^m_r=h_{15}\frac{\partial u^m}{\partial r}-\mu_m\frac{\partial\phi^m}{\partial r},\quad B^m_\theta=\frac{1}{r}\left(h_{15}\frac{\partial u^m}{\partial\theta}-\mu_m\frac{\partial\phi^m}{\partial\theta}\right),\end{aligned}\tag{2}$$

for the PM material. In Eqs. (1) and (2), u is the mechanical displacement; φ and ϕ are the electric potential and magnetic potential, respectively; σ_{iz}, D_i and $B_i (i = r, \theta)$ are the stress, electric displacement and magnetic induction; $G = c_{44}$, e_{15}, h_{15} are the elastic, piezoelectric and piezomagnetic constants; $\kappa = \kappa_{11}$, $\mu = \mu_{11}$ are the dielectric permittivity and magnetic permeability, respectively. The subscripts or superscripts 'e' and 'm' denote the quantities in PE cylinder and PM layer, respectively.

Based on the usual quasi-static electromagnetic approximation, considering the invariant initial stress component T_{rr} which starts from the origin of the core along the r direction, the equilibrium equation with initial stress is [31]

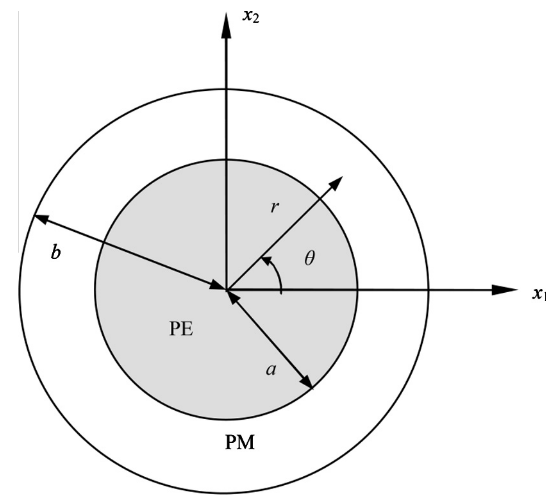

Fig. 1. A PE/PM layered cylindrical structure.

$$\begin{aligned}&\frac{\partial\sigma^e_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma^e_{\theta z}}{\partial\theta}+\frac{1}{r}\sigma^e_{rz}+\left(\frac{\partial^2u_e}{\partial r^2}+\frac{1}{r}\frac{\partial u_e}{\partial r}+\frac{1}{r^2}\frac{\partial^2u_e}{\partial\theta^2}\right)T_{rr}=\rho_e\frac{\partial^2u_e}{\partial t^2},\\&\frac{\partial D^e_{rz}}{\partial r}+\frac{1}{r}\frac{\partial D^e_{\theta z}}{\partial\theta}+\frac{1}{r}D^e_{rz}=0,\\&\frac{\partial B^e_{rz}}{\partial r}+\frac{1}{r}\frac{\partial B^e_{\theta z}}{\partial\theta}+\frac{1}{r}B^e_{rz}=0,\end{aligned}\tag{3}$$

for PE material, and

$$\begin{aligned}&\frac{\partial\sigma^m_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma^m_{\theta z}}{\partial\theta}+\frac{1}{r}\sigma^m_{rz}=\rho_m\frac{\partial^2u_m}{\partial t^2},\\&\frac{\partial B^m_{rz}}{\partial r}+\frac{1}{r}\frac{\partial B^m_{\theta z}}{\partial\theta}+\frac{1}{r}B^m_{rz}=0,\\&\frac{\partial D^m_{rz}}{\partial r}+\frac{1}{r}\frac{\partial D^m_{\theta z}}{\partial\theta}+\frac{1}{r}D^m_{rz}=0,\end{aligned}\tag{4}$$

for PM material.

Here, for the convenience of mathematic derivation and considering the fact that the radial initial stress component is usually much larger than the others in such a cylindrically layered structure, we ignore all the other initial stress components except T_{rr} in calculation. Furthermore, we assume that the initial stress only exists in the PE material and do not consider any direct initial stresses in the PM thin layer. Moreover, the magnitude of the initial electric displacement and magnetic induction due to the initial stress is infinitesimal, so we only consider the influence of the initial stress itself and ignore the effect of the initial electric displacement and magnetic induction caused by the initial stress.

Substituting Eqs. (1) and (2) into Eqs. (3) and (4), we can obtain the governing equations

$$\begin{aligned}&(G_e+T_{rr})\nabla^2u^e+e_{15}\nabla^2\varphi^e=\rho_e\frac{\partial^2u^e}{\partial t^2},\\&e_{15}\nabla^2u^e-\kappa_e\nabla^2\varphi^e=0,\\&\nabla^2\phi^e=0,\end{aligned}\tag{5}$$

for PE material, and

$$\begin{aligned}&G_m\nabla^2u^m+h_{15}\nabla^2\phi^m=\rho_m\frac{\partial^2u^m}{\partial t^2},\\&\nabla^2\varphi^m=0,\\&h_{15}\nabla^2u^m-\mu_m\nabla^2\phi^m=0,\end{aligned}\tag{6}$$

for PM material.