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Bulk longitudinal wave reflection/transmission in periodic piezoelectric structures with metallized interfaces

A.N. Darinskii [a], A.L. Shuvalov [b,c,*], O. Poncelet [b], A.A. Kutsenko [b]

[a] Institute of Crystallography RAS, Leninskii pr. 59, 119333 Moscow, Russia
[b] Univ. Bordeaux, CNRS UMR 5295, I2M-APY, 33405 Talence, France
[c] National University of Science and Technology MISIS, Acousto-Optical Research Center, Leninskii pr. 4, 119049 Moscow, Russia

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ABSTRACT

A theoretical study is performed of the bulk acoustic wave propagation in periodic piezoelectric structures with metallized interperiod boundaries. A crucial specific feature of such structures is that the bounded acoustic beam incident perpendicular to an interface can generate scattered (i.e. reflected and transmitted) waves over the whole area of the interface rather than only within the spot where this acoustic beam crosses the interface as it occurs in the absence of metallization. This extra generation is due to the electric potential which is induced by the incident wave on the whole metallized boundary rather than only on its part. The expressions are obtained for the reflection and transmission coefficients in the case where a longitudinal wave propagates along the 6-fold symmetry axis of a hexagonal piezoelectric. The periodicity is realized by inserting thin metallic layers (electrodes) into otherwise homogeneous material perpendicular to its 6-fold axis. The derived expressions allow the determination of the amplitude of waves arising both inside and outside the incident acoustic beam. The analysis of these expressions shows that the extra generation in question is able to significantly alter the distribution of the wave amplitudes as compared with the pattern which is obtained without taking into account the wave fields appearing outside the domain occupied by the incident acoustic beam.

1. Introduction

Pass- and stop-bands of the frequency spectrum of acoustic waves propagating through periodic structures show up in specific frequency dependences of the reflection and transmission coefficients [1]. The most remarkable feature is that within corresponding frequency intervals the magnitudes of the coefficients vary from values close to zero up to values hardly different from unity. One of the possible practical applications of this effect is the creation of acoustic filters. For instance, surface acoustic wave (SAW) filters are widely utilized in various areas of modern electronics [2,3]. Periodicity in such devices is commonly arranged by depositing a periodic set of metallic strips on the surface of the substrate or by etching a periodic grating of grooves. A SAW being scattered weakly within a single period, long gratings are required to achieve strong effects. Accordingly, SAW filters work in frequency ranges limited from below in order that the size of devices be reasonable. In addition, the stop-band usually occurs to be relatively narrow.

During the last decade there has been a growing interest in producing and investigating acoustic periodic metamaterials, often referred to as phononic crystals (PC), where the transition from almost total reflection to almost total transmission becomes very pronounced for bulk waves [4]. Technology allows the creation of PCs from materials possessing strongly different acoustic properties. Therefore large gaps open in the frequency spectrum and one does not need to use structures involving a good deal of periods to obtain total reflection within stop-bands.

By changing properties of the materials constituting PCs one can control the acoustic properties of PCs, such as the stop-band width and its central frequency, modifying thereby the reflectance and transmittance of the structure within a given frequency range. One can use temperature effects [5,6], electric or magnetic fields [7–9], external mechanical stresses [10,11]. If materials are piezoelectric, then PC parameters can also be tuned by inserting electroconductive patches interconnected by external electric circuits. Patches (electrodes) change electric conditions in the structure, affecting the wave propagation even when they are so thin that

* Corresponding author at: Univ. Bordeaux, CNRS UMR 5295, I2M-APY, 33405 Talence, France.

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the influence of their mechanical properties is negligibly weak. It was demonstrated theoretically that this method allows one to modify dispersion curves of modes guided by two-dimensional PC slabs and piezoelectric beams with electrodes put periodically on the surface [12–14]. In Ref. [15], tunable Bragg band gaps were obtained for a one-dimensional PC whose unit cell is an elastic–piezoelectric bilayer in which the piezoelectric layer has electroded faces connected via external capacitors. The same phenomenon was theoretically and experimentally observed in a homogeneous piezoelectric rod with metallic electrodes inserted perpendicular to the rod axis [16].

In this paper we study theoretically the reflection and transmission of purely bulk, i.e., non-guided, waves propagating perpendicular to metallized interfaces of a periodic piezoelectric structure. It should be said that in considering wave propagation in periodic structure one usually replaces an actual bounded acoustic beam by a plane wave, see, e.g., [10,17,18]. The applicability of this approach is based on the following assumptions. First, diffraction is disregarded. Second, the diameter of the acoustic beam is smaller than the lateral size of the structure, so that the fields are completely concentrated inside the structure, not reaching the exterior lateral border. As a result, the latter has no effect, unlike the propagation of guided modes. The situation becomes more subtle when the interfaces separating layers of the periodic structure are metallized. In this case the electric charges induced on electrodes create potential difference over the whole electrodes. Therefore the corresponding electric field can generate scattered waves not only within the spot where the incident acoustic beam strikes the interface but also in the region lying outside the incident acoustic beam.

If the difference between the cross-sections of the metallized interface and of the incident beam is small enough, then the wave generation outside the incident beam can be disregarded. It is the assumption underlying our considerations carried out in Ref. [19] where we analyzed the influence of an external electric circuit on the one-dimensional propagation of longitudinal wave in piezoelectric periodic composites. The main focus of [19] was on the calculation of effective properties characterizing the homogenized medium.

The present work proposes a simple model allowing one to estimate the contribution of the wave generation outside the space occupied by the incident acoustic beam because of the electric potential induced on the metallized interfaces. The periodic structure is constructed by inserting infinitesimally thin metallic electrodes into a homogeneous piezoelectric. Such a structure allows us to highlight the role of piezoelectricity and simultaneously to simplify evaluations.

The paper is organized as follows. In Section 2 we consider the problem which we conventionally call "the reflection/transmission of a plane wave". The task is to find the reflection and transmission coefficients of a wave having a plane front and filling in with constant amplitude the whole cross-section of the structure. The boundary conditions on the lateral surface are ignored. In other words, here our approach is analogous to that used when studying the plane wave reflection/transmission in periodic structures with non-metallized interfaces [10,17,18]. In Section 3 we consider that an acoustic bounded beam is incident, its diameter being smaller than the size of the structure cross-section and the amplitude being constant over the beam cross-section. We derive the coefficients describing the transformation of the incident wave with account taken of the generation of acoustic waves outside the incident bounded acoustic beam. These coefficients are expressed in terms of the plane wave transformation coefficients obtained in Section 2. Section 4 is devoted to the analysis of the reflection and transmission coefficients found in Section 3. In Section 5 our results are generalized to the case of an acoustic bounded beam with non-uniform amplitude distribution. Section 6 presents the concluding remarks.

2. Plane waves

Let $n+1$ metallic layers (electrodes) be inserted into a piezoelectric medium of symmetry 6 mm perpendicular to the 6-fold symmetry axis at equal distances d from each other forming thereby a periodic structure with n periods. In addition, each two successive electrodes are interconnected by a capacitor of capacitance C. One period of the structure is shown in Fig. 1. The electrodes are so thin that the influence of their mechanical properties on the wave propagation can be disregarded. Only the fact that they are perfect electric conductors plays a role.

We are interested in the reflection and transmission of a longitudinal wave propagating in such a structure along the 6-fold axis (the coordinate z-axis). A possible approach to this sort of problems consists in using the transfer matrix method under assumptions that the incident wave is plane and, accordingly, the structure has an infinite width perpendicular to the direction of wave propagation [17–20]. However, it may be more appropriate to formulate in our case the listed simplifying assumptions as follows: a plane wave whose wave front "takes" an area S is incident on a structure whose cross-section has the same area S. The point is that the area S will enter in expressions involved in our considerations, see Eq. (5) and further.

The elasto-electric field set up in the structure is characterized by the mechanical displacement $u \equiv u_z$ along the z-axis, the mechanical stress $\sigma \equiv \sigma_{zz}$, the electrical potential φ, and the electric displacement $D \equiv D_z$. Forming a 4-component vector-column $\eta = (u, \varphi, \sigma, D)^t$, where the superscript t stands for transposition, one can show that the vectors $\boldsymbol{\eta}_a$ and $\boldsymbol{\eta}_b$ at edges a and b of the piezoelectric layer (Fig. 1) are related by a 4×4 transfer matrix $\hat{\mathbf{M}}$,

$$\boldsymbol{\eta}_b = \hat{\mathbf{M}}\boldsymbol{\eta}_a, \tag{1}$$

where [20]

$$\hat{\mathbf{M}} = \begin{pmatrix} M_{11} & 0 & M_{13} & M_{14} \\ M_{21} & 1 & M_{14} & M_{24} \\ M_{31} & 0 & M_{11} & M_{21} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

with components

$$M_{11} = \cos kd, \quad M_{13} = \frac{1}{\omega Z}\sin kd, \quad M_{14} = \frac{h}{\omega Z}\sin kd, \tag{3}$$

$$M_{21} = h(\cos kd - 1), \quad M_{31} = -\omega Z \sin kd,$$

$$M_{24} = \frac{h^2}{\omega Z}\sin kd - \frac{d}{\varepsilon_{33}},$$

In terms of material constants of the piezoelectric we have

$$k = \omega/v, \quad v = \sqrt{c_{33}^D/\rho}, \quad c_{33}^D = c_{33}^E + e_{33}^2/\varepsilon_{33}, \quad h = e_{33}/\varepsilon_{33}, \quad Z = \rho v, \tag{4}$$

ω is the frequency, v is the longitudinal wave velocity along the 6-fold axis, c_{33}^E is the elastic module at constant electric field, e_{33} and ε_{33} are the piezomodule and the dielectric permittivity, respectively. In deriving Eq. (3) the time factor of the wave field is put $\exp[-i\omega t]$.

Employing the relation [15,16,19]

$$SD = C(\varphi_b - \varphi_a) \tag{5}$$

between the potentials on the electrodes and the electric displacement inside the layer (Fig. 1) allows one to exclude $\varphi_{a,b}$ and D from