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Tuning of band structures in porous phononic crystals by grading design of cells

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ABSTRACT

As the results of the evolution of species, grading structures widely exist in the nature and display distinguish advantages. In this manuscript, grading concept is introduced to redesign the topological structure of pores with the aim to see the effects of grading on the band structure in porous phononic crystals. Circular pores are considered and the crossing grading is made. The wave dispersion in graded structures is investigated comparatively to the normal ones under the same porosity. The band gaps in grading structures are given, as well as the vibration modes of the unit cell corresponding to the absolute band gap (ABG) edges. The results show that the grading structure greatly decreases the critical porosity for the opening of the ABGs. Wider ABGs could be obtained at lower frequencies along with the increase of the porosity. Through controlling the topological parameters of the grading structure, the band structure could be tuned. These results will provide an important guidance in the band tuning in porous phononic crystals by grading design of cells.

1. Introduction

As one kind of novel materials, porous phononic crystals (PPCs), which have periodically distributed open or closed pores in bulk materials, have been widely studied due to the light weight and potential applications in the multi-functional fields, which include mechanical, thermal, and acoustic, etc. [1]. Especially for one important characteristic, the existence of the absolute band gaps (ABGs) [2–4], which represents frequency regions where propagating elastic waves do not exist, attracts more and more attention.

Liu et al. [5,6] investigated the influence of pore shapes, lattice structures and lattice transformation on the band structures in 2D normal porous phononic crystals. Wang et al. [7] discussed the ABG structures in phononic crystals with cross-like holes. Yan and Zhang [8] analyzed the wave localization in two-dimensional porous phononic crystals with one-dimensional aperiodicity. For super phononic crystals, Liu et al. [9] investigated the band gaps in Kagome honeycomb, which is constituted by triangular and hexagonal component cells. Three-dimensional Kagome truss, which is composed by periodically distributed two head-to-head regular tetrahedrons, is also investigated to clarify the band structures in 3D PPCs [10]. These results show that the band structures are sensitive to the pore topological parameters. Through the pore topology design, the band gaps in PPCs could be tuned.

Actually, porous materials widely exist in the nature, from the human bone to the tree trunk. If we observe these natural porous materials carefully, we can see that some are with uniform pores, such as honeycombs, some are heterogeneous, such as bones, depending on the functional or physiological purpose. Researches show that grading structure is one typical topology in natural porous materials, which is the results of natural evolution, and displays noticeably advantageous [11]. In the present discussion, by adopting the concept of grading, we make a try to grade the normal circular porous materials with smaller pores to see the effects of grading on the wave dispersion in porous materials, and hope to provide a new strategy in the pore topological design.

2. Theory

Fig. 1 is a typical representation of a porous phononic crystal with circular pores (denoted by A) periodically arranged in the 2D space, and the z-coordinate is set parallel to the axes of the pores, which are treated as vacuum. Then if the elastic waves propagate in the transverse plane ($x0y$ plane) with the displacement vectors independent of the z-coordinate, they can be decoupled into the mixed in-plane mode and the anti-plane shear mode.

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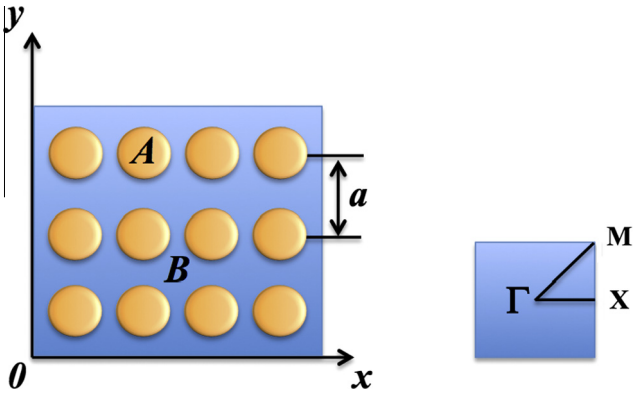

Fig. 1. The transverse cross section of a representative 2D phononic crystal and the first Brillouin zone of the 2D square lattice. a is the lattice constant.

Accordingly, the in-plane wave equations are expressed in the frequency domain as

$$-\rho(\mathbf{r})\omega^2 u_i = \nabla\cdot(\mu(\mathbf{r})\nabla u_i) + \nabla\cdot\left(\mu(\mathbf{r})\frac{\partial}{\partial x_i}\mathbf{u}\right) + \frac{\partial}{\partial x_i}(\lambda(\mathbf{r})\nabla\cdot\mathbf{u}), \quad (i = x, y) \tag{1}$$

In Eq. (1), $\mathbf{r} = (x,y)$ denotes the position vector, ω is the angular frequency, ρ is the mass density, λ and μ are the Lamé constant and shear modulus, $\mathbf{u} = (u_x, u_y)$ is the displacement vector in the transverse plane, and $\nabla = (\partial/\partial x, \partial/\partial y)$ is the 2D vector differential operator. Moreover, the above equation is also available for the solid phononic crystals in which A becomes a solid phase, and the material parameters in Eq. (1) are replaced by the parameters of component A.

According to Bloch theorem, the displacement field can be expressed as

$$\mathbf{u}(\mathbf{r}) = e^{i(\mathbf{k}\cdot\mathbf{r})}\mathbf{u}_\mathbf{k}(\mathbf{r}), \tag{2}$$

where $\mathbf{k} = (k_x, k_y)$ is the wave vector limited to the first Brillouin zone of the reciprocal lattice and $\mathbf{u}_\mathbf{k}(\mathbf{r})$ is a periodical vector function with the same periodicity as the crystal lattice.

In the present discussion, a simple grading structure is adopted. Fig. 2a displays the squarely distributed 2D normal (solid yellow circles) or graded (translucent yellow circles) circular porous phononic crystals. Under the same porosity, a circular pore (Fig. 2b, unit cell of the normal structure) is sub-graded into five circular pores crossing arranged as shown in Fig. 2c (unit cell of the grading structure). The position of the center circular pore, named main pore, is fixed at the center of the unit cell. The sizes of the five pores and positions of the surrounding four circular pores, named minor pores, could be adjusted. This is just one kind of grading strategy. More detailed discussion about grading structure is out of the scope of the present manuscript and will be given in the forthcoming papers.

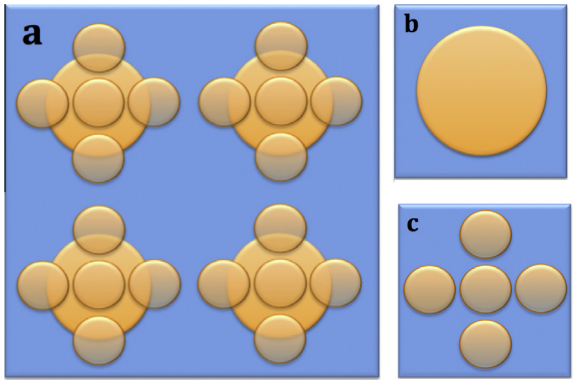

Fig. 2. (a) Diagrammatic sketch of 2D phononic crystals with pores squarely distributed. Solid circles represent the initial structure; (b) unit cell of the normal structure; (c) unit cell of the grading structure.

The Acoustic Module operating under the 2D plane strain Application Mode (ACPN) in COMSOL is applied to solve the governing equations. The discrete form of the eigenvalue equations in the unit cell can be written as

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{U} = \mathbf{0}, \tag{3}$$

where $\mathbf{U}$ is the displacement at the nodes, $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices of the unit cell, respectively. The free boundary condition is imposed on the surface of the pore, and the Bloch boundary conditions on the two opposite boundaries of the unit cell, yielding

$$\mathbf{U}(\mathbf{r} + \mathbf{a}) = e^{i(\mathbf{k}\cdot\mathbf{a})}\mathbf{U}(\mathbf{r}), \tag{4}$$

where $\mathbf{r}$ is located at the boundary nodes and $\mathbf{a}$ is the vector that generates the point lattice associated with the phononic crystals.

Through the maximum cell size and the changing rate controlling, the representative cell is meshed by using the triangular Lagrange quadratic elements provided by COMSOL. Since the present results are compared with the ones given Ref. [9], the detailed convergence analysis of the calculation is not given here anymore. Eigenfrequency analysis is chosen as the solver mode, and the direct SPOOLES is selected as the linear system solver. Moreover, the Hermitian transpose of the constrain matrix and parameter settings in symmetry direction in the advanced solver is required. The model built in COMSOL is saved as a MATLAB-compatible '.m' file. The file is programmed to let the wave vector $\mathbf{k}$ sweep the edges of the irreducible Brillouin zone, so that we can obtain the whole dispersion relations.

3. Numerical examples and discussion

Seen as Fig. 2c, the geometrical parameters of the grading structures could be changed under certain porosity. In this section, we will discuss the effects of the topology variation (size and position) on the band structures in grading PPCs. The results for PPC shown in Fig. 2b are not given here anymore. The detailed results please refer to Ref. [5]. In the calculation, the matrix material is Aluminum, and the pore is vacuum. The elastic parameters used are: λ = 68.3 GPa, μ = 28.3 GPa, and ρ = 2730 kg/m^3. The lattice constant a = 0.2 m.

3.1. Influence of the porosity on the band structures in grading PPCs

For normal PPC shown in Fig. 2b, the critical porosity for the opening of the ABG is f = 0.43 with the porosity given as

$$f = A_{pore}/A_{unit} = n\pi r^2/a^2, \tag{5}$$

where A_{pore} is the area of the pores, and A_{unit} the area of the unit cell. n is the number of the circular pores and r the radius. Keeping this porosity and considering the symmetry of the structure, we are crossing-grading the normal circular pore into five small ones. The centers of the circular pores are then determined (Fig. 2c). Then we change the radii of the pores simultaneously to obtain different porosities from 0.175 to 0.43 by considering the geometrical limitation of the lattice. Of course, other grading strategy could be chosen. The influence of the grading strategy is out of the purpose of this paper and does not discuss here extensively.