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Bulk and surface acoustic waves in solid–fluid Fibonacci layered materials

I. Quotane [a], E.H. El Boudouti [a,b,*], B. Djafari-Rouhani [b], Y. El Hassouani [c], V.R. Velasco [d]

[a] Laboratoire de Dynamique et d'Optique des Matériaux, Département de Physique, Faculté des Sciences, Université Mohamed I, Oujda, Morocco
[b] Institut d'Electronique, de Microélectronique et de Nanotechnologie (IEMN), UMR CNRS 8520, UFR de Physique, Université des Sciences et Technologies de Lille, 59655 Villeneuve d'Ascq, France
[c] EPM, Département de Physique FSTE, Université Moulay Ismail, Boutalamine B.P. 509, Errachidia, Morocco
[d] Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Inés de la Cruz 3, E-28049 Madrid, Spain

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ABSTRACT

We study theoretically the propagation and localization of acoustic waves in quasi-periodic structures made of solid and fluid layers arranged according to a Fibonacci sequence. We consider two types of structures: either a given Fibonacci sequence or a periodic repetition of a given sequence called Fibonacci superlattice. Various properties of these systems such as: the scaling law and the self-similarity of the transmission spectra or the power law behavior of the measure of the energy spectrum have been highlighted for waves of sagittal polarization in normal and oblique incidence. In addition to the allowed modes which propagate along the system, we study surface modes induced by the surface of the Fibonacci superlattice. In comparison with solid–solid layered structures, the solid–fluid systems exhibit transmission zeros which can break the self-similarity behavior in the transmission spectra for a given sequence or induce additional gaps other than Bragg gaps in a periodic structure.

1. Introduction

Phononic crystals (PC) constituted by periodic arrangements (cells) of elastic/acoustic materials according to one (1*D*) [1], two (2*D*) [2], and three (3*D*) [3] dimensions, have been a subject of great interest during the last two decades because of their interesting properties in the development of new acoustical systems [4]. These systems are characterized by the presence of frequency regions where sound can propagate (bulk bands) and frequency regions where sound cannot propagate (gaps). This property has been exploited in the control and the guidance of the propagation of sound in different PCs [5]. As concerns 1*D* systems, different types of periodic structures such as solid–solid and solid–fluid layered materials as well as waveguides with different geometries are conducted as analogs of 2*D* and 3*D* leading to several interesting phenomena such as: omnidirectional band gaps [6,7] and selective transmission by either guided modes [8] or interface resonance modes [9], the possibility to enhance acousto-optical interaction in hypersonic crystals [10,11] and to realize stimulated emission of acoustic phonons [12] as well as ultrasonic metamaterials [13]. The advantage of 1D systems lies in the facilities to design different geometries and they require simple analytical and numerical calculations to understand deeply different physical phenomena observed in such systems.

Besides periodic systems, quasi-periodic ones have been the subject of intensive study during the last two decades [14]. The quasi-periodic structures are generally built from two blocks *A* and *B*. Among them, the Fibonacci structure is constituted following the Fibonacci rule $S_{k+1} = S_k S_{k-1}$ with $S_1 = A, S_2 = AB$ and k is the generation number. This leads to the Fibonacci sequences (FS): $S_3 = ABA, S_4 = ABAAB, S_5 = ABAABABA, \dots$ Merlin et al. [15] were the first to have studied such structure in semiconductor GaAs-AlAs superlattices (SLs). Since this work, much attention has been paid to observe the exotic phenomena of Fibonacci systems [16,17] and interesting characteristics of these systems have been concluded [18] essentially by theoretical studies based on simple 1D models. It is known also that deterministic quasi-periodic systems may exhibit localization, as the Anderson localization, of sound and vibration [19]. Such phenomenon characterize any wave when the structures exhibit disorder [20]. An example of the properties of the propagation and localization of acoustic

* Corresponding author at: Laboratoire de Dynamique et d'Optique des Matériaux, Département de Physique, Faculté des Sciences, Université Mohamed I, Oujda, Morocco.
E-mail address: elboudouti@yahoo.fr (E.H. El Boudouti).

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waves in Fibonacci modulated waveguides have been studied theoretically by some of us [21] and checked experimentally by King and and Cox [22].

Compared to solid–solid SLs, solid–fluid SLs have received less attention [1]. Some years ago [23,24], we have performed an extensive study on the existence of confined and surface modes in finite and semi-infinite SLs made of a periodic repetition of solid–fluid bilayers. We have shown that besides the gaps due to the periodicity of the system these structures exhibit, other gaps due to transmission zeros. These transmission zeros can lead to new phenomena such as acoustic meta-materials [13], Fano resonances [24] as well as the possibility to mimic an acoustic transparency by squeezing a resonance between two transmission zeros [25]. The possibility of using these materials as acoustic mirrors and filters was also demonstrated [24]. Based on this work [24], a recent paper [26] has explored the tunneling effect between two different fluids via a solid–fluid SL with application to petroleum exploration. Also, we have shown the possibility of existence of surface modes in such systems depending on wether the SL is terminated by a solid or a fluid layer [23]. In the long-wavelength limit, an homogenization analysis has been developed [27] to study the effective properties of propagating modes of periodic solid layers in an ideal or viscous fluid.

To our knowledge, few works have been devoted to solid–fluid quasi-periodic structures [28,29]. In these papers finite size FS [28] and periodic FS [29] are studied by means of the transmission coefficient and the localization length deduced from the Lyapunov exponent. The Fibonacci-type structures [14] are formed from two blocks A (fluid layer) and B (solid layer). In reference [28], normal incidence waves through single FS have been considered and the fragmentation of the transmission bands as a function of the generation number has been discussed. However, the self-similarity of the transmission spectra following a scaling law has not been addressed in such systems. In reference [29], a periodic repetition of a FS was considered and the splitting of the bulk bands for normal and oblique incidence was studied by means of the localization length. However, in order to fully characterize the band gap structure of a Fibonacci SL made of a periodic repetition of a given cell where each cell is constructed by a given FS, a dispersion relation involving the Bloch wavevector k and the pulsation Ω should be calculated. In this paper, we give closed form expressions of the transmission coefficient through one FS and the dispersion relation of a given FS repeated periodically. For one FS, we show at normal incidence the property of self-similarity of the transmission spectra each three generations following a scaling law. However, at oblique incidence this property no longer exists because of the existence of transmission zeros. For a Fibonacci SL, we show that when the generation number increases, the pass bands exhibit a fragmentation following a power law. Also we show the existence of different types of gaps: stable and transient gaps induced by the periodicity of the system and new gaps induced by the transmission zeros. These latter gaps are a characteristic of solid–fluid SL and do not exist in solid–solid SL. These bulk properties have not yet been addressed in the literature. In addition to bulk modes we show for the first time the possibility of existence of surface modes in solid–fluid Fibonacci SLs. These modes show different behaviors depending on whether they fall inside stable gaps or transient gaps.

The rest of the paper is organized as follows: in Section 2 we give a brief presentation of the method of calculation employed here, which is based on the Green function method. Section 3 is devoted to the discussion of the numerical results for the transmission along a given Fibonacci sequence for normal and oblique incidence and the dispersion curves for bulk and surface modes in Fibonacci SL. The final section contains the concluding remarks.

2. Method of theoretical and numerical calculation

2.1. Interface response theory of continuous media

In this paper, we consider a multilayered structure made of solid and fluid layers arranged perpendicularly to the x_3 direction. The planes of the layers are contained within the (x_1, x_2) directions. The acoustic waves propagating through such a system are polarized in the sagittal plane defined by the normal to the interfaces (x_3 direction) and the wave vector $k_{//}$ (parallel to the interfaces). We choose $k_{//}$ along the x_1 direction without loss of generality. We consider a non viscous fluid layer for which the viscous skin depth $\sigma = (2\eta/\rho\omega)^{1/2}$ is much smaller than the fluid layer thickness d_f over a very broad frequency range (η and ρ are the viscosity and the density of the fluid). This study is performed with the help of the interface response theory [30] of continuous media which permits us to calculate the Green's function of any composite material. In what follows, we present the basic concepts and the fundamental equations of this theory [30].

Let us consider a composite system defined in its whole space domain labeled D. This system contains different subsystems i connected together by their interface domains M_i. The whole interface space of the system is labeled $M = \cup M_i$. The elements of the Green's function $g(DD)$ of any composite material can be obtained from [30]

$$g(DD) = G(DD) - G(DM)G^{-1}(MM)G(MD) + G(DM)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD), \quad (1)$$

where $G(DD)$ is the Green's function of a given continuous medium and $g(MM)$ is the Green's function of the composite system in its interface domain M. As we are interested in elastic waves in solid and fluid media, the corresponding Green's functions $G(DD)$ can be derived from the equation of motion of displacement fields as explained in Ref. [30]. The inverse $[g(M,M)]^{-1}$ of $g(MM)$ is obtained as a superposition of the different $g_i^{-1}(M_i, M_i)$ [30], inverse of the $g_i(M_i, M_i)$ for each constituent i of the composite system.

The inverse of $g(MM)$ enables us to obtain the eigenmodes of a composite system through the relation [30]

$$det[g^{-1}(MM)] = 0, \quad (2)$$

$U(D)$ being an eigenvector of the reference system, Eq. (1) leads to the eigenvectors $u(D)$ of the composite material as

$$u(D) = U(D) - U(M)G^{-1}(MM)G(MD) + U(M)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD). \quad (3)$$

In Eq. (3), $U(D), U(M)$, and $u(D)$ are row vectors. If $U(D)$ is a bulk wave launched in one homogeneous piece of the composite material, then Eq. (3) enables the calculation of all the waves reflected and transmitted by the interfaces, as well as the reflection and transmission coefficients of the composite system [1].

2.2. Inverse surface Green's functions of the elementary constituents

As mentioned above, the calculation of the Green's function of any composite material made of solid and fluid layers, within the interface response theory, requires the knowledge of the surface elements of its elementary constituents, namely, the Green's function of an ideal fluid of thickness d_f, sound speed v_f and mass density ρ_f and an elastic isotropic solid characterized by its thickness d_s, longitudinal speed v_ℓ, transverse speed v_t, and mass density ρ_s. In addition, the calculations of the dispersion relations and