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Propagation of flexural waves in inhomogeneous plates exhibiting hysteretic nonlinearity: Nonlinear acoustic black holes

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ABSTRACT

Theory accounting for the influence of hysteretic nonlinearity of micro-inhomogeneous material on flexural wave in the plates of continuously varying thickness is developed. For the wedges with thickness increasing as a power law of distance from its edge strong modifications of the wave dynamics with propagation distance are predicted. It is found that nonlinear absorption progressively disappearing with diminishing wave amplitude leads to complete attenuation of acoustic waves in most of the wedges exhibiting black hole phenomenon. It is also demonstrated that black holes exist beyond the geometrical acoustic approximation. Applications include nondestructive evaluation of micro-inhomogeneous materials and vibrations damping.

1. Introduction

From the fundamentals point of view the interest to the nonlinear acoustic waves in general and in particular to those localized in their propagation near the surfaces, interfaces and wedges is continuous [1–3]. At the same time this interest is fueled by existing or emerging applications of the wedge, surface and plate acoustic waves such as vibrations damping [4], nondestructive testing of blades [5], material characterization [6] or biomedical [7], for example. There are always such usual questions to be answered, which are accompanying any application of the acoustic waves, as: "Are the nonlinear acoustic effects important in the considered phenomena?" and "Could the nonlinear acoustic effects be useful in the applications"?

In this Letter we develop the simplest asymptotic theory describing the propagation of the plane nonlinear flexural waves in the plates of the variable thickness. Our particular attention will be given to the analysis of the plates with thickness variations following a power law $(H(x) = H(x_0)(x/x_0)^{\alpha} \equiv H_0\xi^{\alpha}, \alpha \geqslant 0$, where H denotes the thickness, x_0 stands for a particular x-coordinate, where the plate thickness takes the value $H_0 \equiv H(x_0)$, and $\xi \equiv x/x_0$ is the normalized coordinate). Our interest to the investigation of the nonlinear phenomena in plane flexural waves propagating along the x-axis toward the edge, in $\xi = 0$, of the above described "power-law" wedge was motivated by several different factors.

Firstly, on the basis of both the simplest geometrical considerations for elastic wave energy concentration and the existing theories [8–10] it could be expected that the amplitude of the monochromatic flexural wave should grow infinitely in elastically linear ideal, i.e., lossless, power-type wedge with $\alpha > 0$ when the wave is approaching the wedge edge. This is a clear indication that the nonlinear phenomena could be important or even unavoidable under particular circumstances, and that the development of the nonlinear theory is desirable.

Secondly, quite recently there was a revival of the interest to the flexural acoustic black holes in view of their application as effective absorbers of the vibrations [4,10,11]. The considered black hole effect is based on the theoretical prediction of the infinite time needed for the flexural waves in the wedges with $\alpha \geqslant 2$ to propagate from any point of the plate toward its edge in $\xi = 0$. Thus, the wave travelling in the direction of the edge never reaches it, is never scattered by it, and never gives the information in the form of the reflected wave on its existence in $\xi = 0$. At the same time the backscattering of the wave in each point of the plate, which could be potentially expected because of the plate spatial inhomogeneity, could be also negligibly small for the waves of short lengths propagating in the plates of slowly varying thickness, when the conditions of the so-called geometrical acoustic approximation

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[10,12–14] are satisfied. Consequently, the wave incident on the black hole-type wedge is not reflected and its energy could be dissipated in the wedge via even weak mechanisms of linear acoustic absorption [4,10]. The efficiency of this emerging technique of vibrations damping can be enhanced for applications by increasing the linear acoustic absorption artificially [4]. However the question on a possible role of the nonlinear acoustic absorption in the black hole phenomena has not been studied yet to our knowledge. There are multiple classic [15–18] and modern [19–28] studies of the nonlinear acoustic absorption (nonlinear "internal friction" in classical terminology) in various kind of micro-inhomogeneous [20,26,29] materials, where the important role in the mechanical motion is played by dislocations, grain boundaries, inter-grain contacts, cracks, etc., i.e., by the mechanical elements that are significantly larger in dimensions than interatomic distances in the sample but, at the same time, are significantly smaller than the sample dimensions. These materials are also called mesoscopic [30,31] from the acoustic point of view, when they are tested by the acoustic waves which are significantly exceeding in length the dimensions of the mechanical elements responsible for their inhomogeneity. It is well documented in the literature that the nonlinear absorption can dominate over the linear one in the acoustic resonance experiments, i.e., in standing waves [23,25,26,28,32,33], causing the increase in the width of the resonance significantly beyond the linear resonance width. However, important nonlinear absorption was also revealed in the travelling acoustic waves both in the microstructured and nanostructured materials [21,22,27,34,35]. Thus, the accumulated evidence of the nonlinear acoustic absorption in micro-inhomogeneous materials indicates that it is worth studying the nonlinear acoustic absorption of flexural waves in the micro-inhomogeneous plates in view of their potential applications for vibrations damping.

Thirdly, the studies of the amplitude-dependent flexural wave phenomena could potentially provide easier access to the evaluation of some particular fundamental types of the materials nonlinearity. In fact, the symmetry of the pure flexural waves, i.e., of the pure bending motion [36], dictates that the nonlinearity effecting flexural waves should be of odd-type. For examples classical quadratic elastic nonlinearity (of even-type in its symmetry and quadratic in its dependence on wave amplitude), which in homogeneous materials is due both to the nonlinearity of the kinematic/geometric relation between the strain and the displacement gradients (kinematic/geometric nonlinearity) and due to the nonlinearity of the elastic stress–strain relationship (physical nonlinearity) [29] should not influence the propagation of flexural waves by symmetry considerations. This situation is similar to one with plane bulk shear acoustic waves [29,37,38]. The lowest order nonlinearity which is currently under the consideration for flexural motion in homogeneous materials is classical cubic elastic nonlinearity (of odd-type in its symmetry and cubic in its dependence on the wave amplitude) [39–41]. The theories of flexural waves, which include the classical elastic nonlinearities only, could be not relevant for the flexural waves propagation in micro-inhomogeneous plates where other types of nonlinearities, i.e., non-classical nonlinearities, could dominate [28,31]. Of particular importance could be the so-called hysteretic quadratic nonlinearity (HQNL) of odd-type in its symmetry and quadratic in its dependence on the wave amplitude [29,32,37,42], which is a paradigmatic example, even though the nonlinear absorption and dispersion mechanism by hysteretic micromechanical units is generic. Due to its odd symmetry HQNL can influence the flexural wave propagation, while due to its quadratic dependence on the wave amplitude it could dominate over the classical elastic cubic nonlinearity in the case of weakly nonlinear wave phenomena in flexural waves. Moreover the HQNL contains both elastic and inelastic parts [29,32,37,42]. The latter should lead to direct nonlinear absorption of flexural waves, while the classical cubic nonlinearity is purely elastic and does not directly contribute to acoustical absorption. Note, that the processes of higher harmonics generation due to elastic nonlinearity, which could lead to the depletion of energy in the flexural wave at fundamental frequency, are importantly suppressed in flexural waves because of the inherent dispersion of flexural wave velocities [36] preventing synchronous wave interactions. All this motivates the studies of the flexural waves in plates exhibiting non-classical nonlinearities. It could be expected that experiments with flexural waves would provide in perspective easier access to the fundamental evaluation of the non-classical material nonlinearities, because of the suppression of classical quadratic elastic nonlinearity by system symmetry, and would become useful in the nondestructive testing of micro-
inhomogeneous materials.

The manuscript is structured as follows. In Section 2 the existent knowledge on the flexural waves in the plates of the variable thickness, revealed in the GA approximation [12,13], is briefly reviewed. In Section 3, exploiting GA and rotating phase [43,44] approximations, the theory for the flexural waves in the plates of inhomogeneous thickness exhibiting HQNL is developed. In Section 4 we discuss the possible extensions of the obtained theoretical prediction for the plates exhibiting other types of hysteretic nonlinearities [24,45,46] or other types of the nonlinear absorption [23,46]. In Section 5 we provide instructive examples, demonstrating that the black hole effect in flexural waves exists beyond the GA approximation. The conclusions are presented in Section 6.

2. Linear flexural waves

A classical derivation of the equation for plane bending/flexural wave propagation [32] combines the equation for the vertical displacement $u_z(t,x)$ of the plate

$$\rho H(x)\partial^2 u_z/\partial t^2 = \partial Q_x/\partial x, \tag{1}$$

where ρ is the plate density, $H(x)$ is its thickness (Fig. 1), with the relation of the shear force per unit length $Q_x(t,x)$ to the spatial derivatives of the displacement field $u_z(t,x)$. The latter is established by using the geometric relation of normal strain $\partial u_x(t,x,z)/\partial x$ with the curvature of the plate

$$\partial u_x/\partial x = -z\partial^2 u_z/\partial x^2 \tag{2}$$

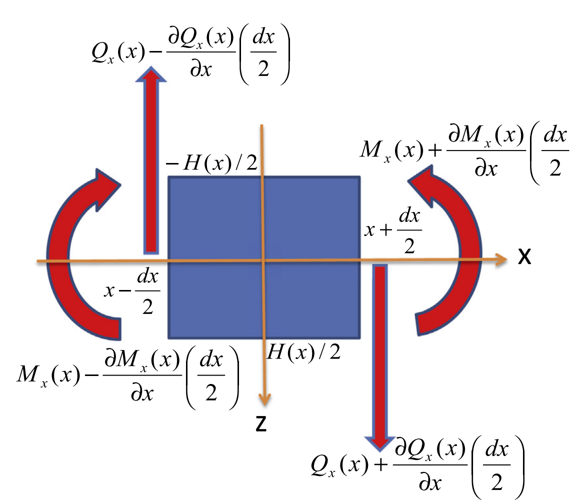

Fig. 1. An element of a plate of local thickness $H(x)$ and of infinitesimal length dx, showing the forces and moments acting on it from the surrounding material of the plate [32]. The flexural waves are assumed to travel in the $-x$ direction.