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An extended diffraction tomography method for quantifying structural damage using numerical Green's functions

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ABSTRACT

Existing damage imaging algorithms for detecting and quantifying structural defects, particularly those based on diffraction tomography, assume far-field conditions for the scattered field data. This paper presents a major extension of diffraction tomography that can overcome this limitation and utilises a near-field multi-static data matrix as the input data. This new algorithm, which employs numerical solutions of the dynamic Green's functions, makes it possible to quantitatively image laminar damage even in complex structures for which the dynamic Green's functions are not available analytically. To validate this new method, the numerical Green's functions and the multi-static data matrix for laminar damage in flat and stiffened isotropic plates are first determined using finite element models. Next, these results are time-gated to remove boundary reflections, followed by discrete Fourier transform to obtain the amplitude and phase information for both the baseline (damage-free) and the scattered wave fields. Using these computationally generated results and experimental verification, it is shown that the new imaging algorithm is capable of accurately determining the damage geometry, size and severity for a variety of damage sizes and shapes, including multi-site damage. Some aspects of minimal sensors requirement pertinent to image quality and practical implementation are also briefly discussed.

1. Introduction

Structural health monitoring (SHM) technologies hold significant potential for improving the design and management of high-value structural assets, such as aircraft components, by enhancing structural reliability and reducing inspection costs. With the recent significant progress towards quantitative damage imaging through the application of time-reversal imaging [1,2], it is now possible to obtain quantitative characterisation of damage, including location, shape and size, even in a stiffened plate-like structure with complex features. However, the current implementations of the imaging algorithms in [1,2] are incapable of predicting the damage severity, such as changes in thickness or in flexural stiffness, which is required in estimating the residual strength and fatigue life of the structure. Plate-wave diffraction tomography as defined in [3] can reconstruct the damage severity, but only under far-field conditions in which the distance between the sensors and damage is greater than $2d^2/\lambda$ [4], where d is the diameter of the damage and λ is the wavelength. For practical applications of quantitative SHM and prognostics techniques, a near-field imaging algorithm is required. The aim of this paper is to extend the work in [3,5] to near-field imaging of laminar damage by flexural waves. The validity and accuracy of this new algorithm are assessed using synthetic data for laminar damage in an isotropic plate for a variety of damage shapes and sizes, including near-field imaging and multi-site damage; an example using experimental data is also included.

Guided wave based SHM methods have attracted considerable interest as an efficient means for wide-area damage detection and characterisation, as the waves employed to interrogate the structure can be tuned to propagate in appropriate modes to ensure optimum sensitivity to the anticipated damage mode and size [6,7]. Wang et al. [8,9] were the first to develop a time-reversal imaging concept to locate damage using the scattered wave signals captured by an array of active sensors. It was shown that the time-reversal method in the time-domain is efficient in detecting and locating damage. The reverse time migration technique, widely employed in geophysical exploration, has been adapted to detect and reconstruct an image of damage in plates by Lin and Yuan [10]. This scheme, using a linear array of piezoelectric sensors, was verified with numerical simulations for damage in an isotropic plate. Zhao et al. [11] developed a novel reconstruction algorithm for probabilistic inspection of defects which is based on correlation analysis of the baseline and scattered guided-wave signals. The

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concept was experimentally demonstrated with piezoelectric sensors bonded onto the surface of an aircraft aluminium wing and a resulting probability map was constructed to characterise the damage.

Michaels and Michaels [12] applied time-shift algorithms to experimentally-measured scattered wave signals to locate defects in an aluminium plate. The resulting multiple images were stitched together as a single image to reduce background signal noise and depict the location of the damage graphically. Employing a similar concept of superimposing a sequence of images of the same structural integrity state, Su et al. [13] presented an imaging procedure through experimental work for the detection of damage in composites based on scatter wave signal time-of-flight and the resulting probability densities of damage occurrences. Ng and Veidt [14] proposed an imaging method based on a digital beam forming technique. Individual sensor–actuator pair images created from cross-correlation of the scattered signal and excitation pulse were combined to predict the location of the damage. Other similar approaches have been briefly reviewed in [5].

To achieve the more ambitious task of determining damage size and severity, there have been several attempts at tomographic imaging based on Lamb waves. The most common approach has aimed to image variations in wave speed, with the objective of converting these into images of thickness variations by using the dispersion relations for wave speed as a function of frequency-thickness. Early work was based on the assumption of straight-ray propagation, which does not correctly capture the physics of plate wave interaction with damage, resulting in poor quality images [15,16]. Malyarenko and Hinders [17] reviewed several papers from medical and geophysical contexts that described a two-step approach to imaging, with the first step being to reconstruct a coarse, low-resolution image based on bent-ray travel-time (or time-of-flight) tomography, followed by a second step based on some form of diffraction tomography (or wave field inversion) to obtain a higher resolution image. These authors provided an implementation of the first step (bent-ray tomography) for Lamb waves, but not of the second step. More recent implementations of straight-ray tomography have been presented in [18,19] and references therein.

A theoretical framework for plate-wave diffraction tomography was first derived in [20,21], based on Mindlin plate theory, which provides an accurate characterisation of A_0 mode at frequencies below the cut off for the A_1 mode. This framework was implemented by Rhode et al. [22,23], using both experimental and computational data. The theoretical framework was extended in [3] to derive an analogue of the filtered back propagation (FBP) algorithm [24] plate-wave diffraction tomography. Belanger et al. [25] implemented an alternative approach based on the theoretical relationship derived in [26] between far-field beamforming and diffraction tomography, and demonstrated the approach using both computational and experimental data. Huthwaite and Simonetti [27] have presented a two-step procedure for high-resolution guided-wave imaging based on (i) bent-ray travel-time tomography for the first step, and (ii) using the relation between beamforming and diffraction tomography for the second step. This approach extends their previous work on breast imaging [28] and it is based on an acoustic model for wave propagation and scattering. More recently, Huthwaite [29] has presented an extensive characterisation of the imaging performance obtained by this approach, using computational data obtained from both (i) an acoustic model, and (ii) a more realistic finite element (FE) model, for the scattered field data. It was found that in the latter case, the resolution achieved is around $1.5 - 2\lambda$, instead of the resolution of $\lambda/2$ which is achieved for the acoustic data, and which would be expected theoretically [28]. This lower resolution was attributed to the inadequacy of the acoustic model in correctly describing guided-wave scattering in elastic plates, based on the observed difference between the scattering pattern obtained from the FE model relative to that from the acoustic model for a small point-like defect (diameter = $\lambda/4$).

This paper differs from the previous work outlined above in several respects. First, it is noted that the filtering operation in the spatial-transform domain that is derived and implemented in [25–29] is unnecessary, because the FBP algorithm directly produces the same result in one step. This equivalence assumes far-field insonification and reception, as further discussed in [5]. Secondly, it has been shown in [3] how the FBP algorithm can be adapted to plate-wave imaging by incorporating the correct form for the point-scatterer directivity pattern. Finally, it is also shown in [5] how the FBP algorithm can be modified to accept as input the multi-static data matrix representing the scattered field at a point-like receiver (or sensor) due to a unit-amplitude excitation from a point-like source (actuator). The resulting algorithm combining these features is presented and evaluated for the first time in the present work. In addition, the imaging algorithm requires the Green's function, which is the undamaged structure's response to a unit input. This response is not available analytically for complex structures, but it can be determined computationally, using for example the FE method, as demonstrated in the present work.

The paper is organised as follows. Section 2 introduces the imaging problem i.e., the measurement set up and the physical property to be measured and inverted for imaging. Section 3 presents the new algorithm, based on an extension of [3]. Section 4 discusses the FE model setup, and calculation of the numerical Green's function datasets as well as calculation of the multi-static data matrix for a laminar damage. The term laminar damage refers to forms of damage that extend parallel to the plate's mid-plane, rather than at right angles to it; examples are corrosion thinning, exfoliation corrosion and delaminations in fibre-composite laminates. Section 5 presents the imaging results for four numerical examples, including various shapes and multi-site damage, and Section 6 presents results for experimental data. Finally, Section 7 discusses the significance of the results and indicates some generalisations of the new imaging method developed in this paper towards the development of a generic framework for imaging laminar damage in complex structures.

2. Problem formulation

Consider an isotropic material i.e., aluminium plate-like structure equipped with a network of actuators, at position $\boldsymbol{X}_s$, along a closed curve Γ_s, and receivers at locations $\boldsymbol{X}_r$, along a possibly different curve Γ_r [5]. For simplicity, discrete active sensors that serve the dual roles of actuators and receivers are considered herein, and these sensors are assumed to be distributed along a single closed curve Γ, ($\Gamma_s = \Gamma_r$) that encloses a specified imaging domain which is indicated as a grey area in Fig. 1. It is assumed that the structure has suffered some form of laminar damage that lies within this imaging domain, as illustrated by the shadowed region denoted by Σ_d in Fig. 1.

For simplicity, the laminar damage is modelled here as a localised reduction in plate thickness δ_h, which is representative of corrosion thinning, except that the thinning is assumed to be symmetric with respect to the plate's mid-plane, so as to avoid mode coupling with the symmetric modes. A further extension of the algorithm that employs the mode-coupled scattered field has been discussed in [30]. However, for moderate thickness reductions, a relatively small fraction of the scattered energy resides in the mode-converted waves, so that good images can be obtained by considering only the non-mode-converted scattered field, as shown in [25,27]. The plate is excited by a vertical force at a