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Investigation of a cup-shaped ultrasonic transducer operated in the full-wave vibrational mode

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ABSTRACT

Cup-shaped horn has significant applications in ultrasonic machining, such as continuous bonding of plastic sheet or strips. Generally, it is excited by a sandwich piezoelectric transducer and both together constitute a cup-shaped ultrasound transducer (CUT). To provide a concise theoretical model for its engineering applications, the equivalent circuit of the cup-shaped ultrasonic transducer is deduced and the resonance/anti-resonance frequency equations are obtained. Meanwhile, the vibrational characteristics of the cup-shaped ultrasonic transducer have been investigated by using the analytical and numerical methods, and then confirmed by the experiment. The results show that the cup-shaped horn has a distinctive equivalent circuit, and the cup-shaped ultrasonic transducer has a good vibrational performance.

1. Introduction

In ultrasonic applications where high displacements are required, the output displacement of the ultrasonic transducer has to be amplified by means of a mechanical structure commonly called ultrasonic concentrator or ultrasonic horn. Among possible ultrasonic horns, sectional ultrasonic horns, made from rods of variable cross section, such as exponential, conical, catenoidal, stepped, or more complex, are those that have been mainly exploited in applications [1,2].

However, for applications to processes such as continuous seam welding of plastic sheet or strips, disk type ultrasonic cutting and ultrasonic drilling, the tubular type ultrasonic tools are needed [3,4]. For the ultrasonic tubular tools or radiators with constant cross-sectional areas [5,6], because they cannot directly amplify the output displacements of the ultrasonic transducers, the displacement amplifiers are needed between the transducers and the tools when the higher displacements are required, which makes the ultrasonic vibration system more complex.

In order to improve the vibrational performance of the ultrasonic transducers used in the applications mentioned above, a cup-shaped ultrasonic transducer (CUT) is proposed in this paper. It is composed of a symmetrical sandwich piezoelectric actuator and of a cup-shaped horn working not only as a displacement amplifier but as an ultrasonic tool. The equivalent circuit of the CUT is deduced and its resonance/anti-resonance frequency equations are obtained. The vibrational characteristics of the transducer are investigated by analytical and numerical methods, and then measured experimentally. The results show that the CUT has high amplitude of the operating mode, uniformity of amplitude at the working surface and better isolation of the operating frequency from close non-tuned modes.

2. Equivalent circuit and resonance frequency equation

Fig. 1 schematically shows the proposed transducer: It is composed of a symmetrical sandwich piezoelectric actuator and of a cup-shaped horn. The sandwich piezoelectric actuator is composed of two couple of piezoelectric rings with inner radius r_a, outer radius r_b and thickness t (the total length of the piezoelectric stack is $L_0 = n \cdot t$) and of two cylinder shaped steel masses, which have a radius identical to the outer radius of the piezoelectric rings and length, i.e. $L_b = L_f$. The cup-shaped horn is used to amplify the vibration amplitude of the piezoelectric actuator. The amplification factor depends upon the ratio between the back and the front cross-sectional area of the horn.

2.1. The equivalent circuit of the symmetrical sandwich piezoelectric actuator

The symmetrical sandwich piezoelectric actuator shown in Fig. 1 can be broken down into three general parts, i.e. the piezoelectric rings and the stress blot, the back cylinder shaped steel mass and the front cylinder shaped steel mass. For the piezoelectric rings and the stress bolt, they are modeled using the improved

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Nomenclature

r_a	inner radius of the piezoelectric rings
r_b	outer radius of the piezoelectric rings
r_S	radius of the stress bolt
r_1	outer radius at the input cross-sectional area of the cup-shaped horn
r_2	inner radius at the input cross-sectional area of the cup-shaped horn
r_3	outer radius at the output cross-sectional area of the cup-shaped horn
t	thickness of each piezoelectric ring
n	number of piezoelectric ceramic elements
L_0	($L_0 = n \cdot t$) the total length of the piezoelectric stack
L_b	length of back steel mass of the sandwich piezoelectric actuator
L_f	length of front cylinder shaped steel mass of the sandwich piezoelectric actuator
L	length of the cup-shaped horn
$\alpha = \frac{r_3 - r_2}{r_2 \cdot L}$	variation coefficient of the cross-sectional radius
S_0	cross-sectional area of the piezoelectric ceramic elements
S_S	cross-sectional area of the stress bolt
S_b	cross-sectional area of the back steel mass of the sandwich piezoelectric actuator
S_f	cross-sectional area of the front steel mass of the sandwich piezoelectric actuator
S_1	input cross-sectional area of the cup-shaped horn
S_2	output cross-sectional area of the cup-shaped horn
ρ_P	density of the piezoelectric elements
ρ_S	density of the stress bolt
ρ_b	density of the back steel mass of the sandwich piezoelectric actuator
ρ_f	density of the front steel mass of the sandwich piezoelectric actuator
ρ	density of the cup-shaped horn
v_P	longitudinal velocity of sound for the piezoelectric stack
v_S	longitudinal velocity of sound for the stress bolt
v_b	longitudinal velocity of sound for the back steel mass of the sandwich piezoelectric actuator
v_f	longitudinal velocity of sound for the front steel mass of the sandwich piezoelectric actuator
v	longitudinal velocity of sound for the cup-shaped horn
k_P	longitudinal wavenumber ω/v_P
k_S	longitudinal wavenumber ω/v_S
k_b	longitudinal wavenumber ω/v_b
k_f	longitudinal wavenumber ω/v_f
k	longitudinal wavenumber ω/v
ω	angular frequency $2\pi f$
j	$(-1)^{\frac{1}{2}}$
S_{ij}^E	elastic compliance constant
d_{31}	piezoelectric strain constant
k_{33}	electro-mechanical coupling coefficient
ε_{33}^T	dielectric constant measured at constant stress
C_0	clamped capacitance for the piezoelectric ceramic stack
N	electromechanical transformation coefficient
Z_{0S}	characteristic impedance for stress bolt $\rho_S v_S S_S$
Z_{0P}	characteristic impedance for the piezoelectric ceramic elements $\rho_P v_P S_0$
Z_{0f}	characteristic impedance for the front steel mass of the sandwich piezoelectric actuator $\rho_f v_f S_f$
Z_{0b}	characteristic impedance for the back steel mass of the sandwich piezoelectric actuator $\rho_b v_b S_b$
Z_{01}	characteristic impedance at the input end of the cup-shaped horn $\rho v S_1$
Z_{02}	characteristic impedance at the output end of the cup-shaped horn $\rho v S_2$
Z_{iS}	mechanical impedance in the equivalent circuit of the stress bolt $\rho_S v_S S_S$
Z_{iP}	mechanical impedance in the equivalent circuit of the piezoelectric ceramic elements
Z_{if}	mechanical impedance in the equivalent circuit of the front steel mass of the sandwich piezoelectric actuator
Z_{ib}	mechanical impedance in the equivalent circuit of the back steel mass of the sandwich piezoelectric actuator
Z_{11}	open-circuit impedance in the equivalent circuit of the cup-shaped horn
Z_{12}	open-circuit impedance in the equivalent circuit of the cup-shaped horn
Z_{21}	open-circuit impedance in the equivalent circuit of the cup-shaped horn
Z_{22}	open-circuit impedance in the equivalent circuit of the cup-shaped horn
ξ	longitudinal displacement amplitude of the cup-shaped horn
ξ_1	longitudinal displacement amplitude at the input surface of the cup-shaped horn
ξ_2	longitudinal displacement amplitude at the output surface of the cup-shaped horn
M_p	amplification factor $\left\lvert \frac{\xi_2}{\xi_1} \right\rvert$
$\dot{\xi}_1$	longitudinal vibrational velocity at the input surface of the cup-shaped horn
$\dot{\xi}_2$	longitudinal vibrational velocity at the output surface of the cup-shaped horn
F_1	longitudinal force at the input surface of the cup-shaped horn
F_2	longitudinal force at the output surface of the cup-shaped horn
E	Young's modulus
$[\mathbf{C}]$	elasticity matrix of the piezoelectric material
$[\mathbf{e}]$	piezoelectric stress matrix
$[\varepsilon]$	dielectric relative permittivity matrix at constant strain
Z_{Ci}	input mechanical impedance for the cup-shaped horn
Z_{bi}	input mechanical impedance for the back metal mass
Z_{mi}	input mechanical impedance of the transducer
Z_e	input electro-mechanical impedance of the transducer
V	electric potential
I	current

Mason's equivalent circuit for a length-extensional resonator [7–9]. The back cylinder shaped steel mass and the front cylinder shaped steel mass are modeled using two T-networks, respectively. The equivalent circuit for the assembly of the three general regions in the symmetrical sandwich piezoelectric actuator is shown in Fig. 2. The parameters of the equivalent circuit for the piezoelectric actuator shown in Fig. 2 are: $C_0 = \frac{n^2 \varepsilon_{33}^T S_0}{L_0}(1 - k_{33}^2)$, $N = \frac{n S_0 d_{33}}{L_0 S_{33}^E}$, $Z_{1S} = jZ_{0S}\tan\left(\frac{k_s L_0}{2}\right)$, $Z_{1P} = jZ_{0P}\tan\left(\frac{k_P L_0}{2}\right)$, $Z_{2S} = \frac{Z_{0S}}{j\sin(k_s L_0)}$, $Z_{2P} = \frac{Z_{0P}}{j\sin(k_P L_0)}$, $Z_{1b} = jZ_{0b}\tan\left(\frac{k_b L_b}{2}\right)$, $Z_{2b} = \frac{Z_{0b}}{j\sin(k_b L_b)}$, $Z_{1f} = jZ_{0f}\tan\left(\frac{k_f L_f}{2}\right)$, $Z_{2f} = \frac{Z_{0f}}{j\sin(k_f L_f)}$. Where $Z_0 = \rho v S$, ρ, v and S are the density, velocity and cross-sectional area of the mechanical element; $k = \frac{\omega}{v}$ is the wave number. The subscripts designate whether the value corresponds to the stress bolt S, the piezoelectric stack p, the back cylinder shaped steel mass b or the front cylinder shaped steel mass f.