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A decimated minimum variance beamformer applied to ultrasound imaging

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A R T I C L E I N F O

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A B S T R A C T

Minimum variance beamforming has performed significant improvement in the resolution of the ultrasound images. However, its computational complexity is a serious problem. This paper introduces a new implementation of the minimum variance beamformer for ultrasound imaging with a focused transmit beam. In this method, a decimated aperture data instead of full of it, is used as the beamformer input, on which the minimum variance beamforming is applied, with the covariance matrix estimated using the full aperture data. In this way, the method can give a linear complexity while it can show a performance comparable to that of the full array implementation of the minimum variance beamforming, as the simulation and experimental results confirm this. Therefore, this adaptive beamforming method can be viewed as an approximate implementation of the minimum variance beamforming with a linear computational complexity.

1. Introduction

Medical ultrasound imaging is conventionally done through a delay and sum (DAS) beamformer. This data independent beamformer applies a preset apodization on the signals transmitted or received by the array elements to control the beampattern properties. In contrast, data dependent adaptive beamformers update the apodization vector for each point of image, so that the signals received from the points far from the point of interest are lowered.

The minimum variance (MV) beamforming is one of the most widely studied adaptive beamforming methods for ultrasound imaging [1–6], which has shown significant improvement in the resolution of the ultrasound images. In spite of its capability in the enhancement of the image, the computational complexity is a major drawback of it, which makes its application in real time ultrasound imaging a serious problem. While the conventional delay and sum (DAS) beamformer requires the computation load at the order of N (i.e. $\mathcal{O}(N)$) at each time, that of the MV is $\mathcal{O}(N^3)$, where N is the number of array elements. This complexity is a cost paid for the enhancement achieved by the MV method.

One way proposed to reduce the computational complexity of the MV is the beamspace domain MV beamformer [7,8]. This method employs the fact that the signals received from the points far from the focal points are very small and negligible, as a result of the focusing applied on the transmission. In this way, a good approximate implementation of the MV beamforming is reached. In [9], the authors proposed a method to obtain a covariance matrix in a Toeplitz structure, based on which the inverse of the matrix can be computed using a fast algorithm. Both of these methods reduce the complexity to $\mathcal{O}(N^2)$, while performing a performance comparable to that of MV.

Synnevag et al. [10] have proposed a low complexity adaptive (LCA) beamforming method based on the MV beamforming idea with a complexity of $\mathcal{O}(N)$. The method is based on the evaluation of the beamformer output for a set of predefined apodization and applying the best one. The evaluation criterion was an approximate value of the output power acquired through a temporal averaging. The method gives significant improvement in image resolution compared with the DAS. To retain the complexity to $\mathcal{O}(N)$, they have ignored the spatial averaging and this restricts its capability to enhance the resolution. Moreover, the window design should be carefully done and may be application-dependent. The LCA also shows some artifacts on the image because of the discrete solution space used.

This paper describes a new method to reduce the complexity of the MV. This method named as decimated minimum variance (DMV) beamformer uses the same fact used in the beamspace MV [7]; but applying it in array domain, instead of transform domain (beamspace). The key feature of the method is the decimation of the array without significant degradation of the obtained image. This feature reduces the number of weights which are adaptively calculated; however the covariance matrix is estimated through using all data received by the array. This property distinguishes the new method from applying the MV on a sparser array.

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In this way, the DMV beamformer is an approximate implementation of the MV and is capable to reduce the complexity to $\mathcal{O}(N)$, even with spatial and temporal averaging.

The paper is organized as follows. The next section describes the new method through mathematical expression. Section 3 is devoted to the way of estimating the covariance matrix and then some designing considerations are discussed in Section 4. The simulation results are given in Sections 5, including some discussions on resolution, contrast and robustness properties and the comparison with the DAS and MV methods. The performance of the method on the experimental data is presented in Section 6 and concluding remarks are given in Section 7.

2. The method

In array imaging systems, echo signals from the focal point arrived at individual transducer array elements are coherent and those emanating from the points far from the focal point are incoherent. It means that in the spatial frequency domain, the low frequency components are related to the signals received from the focal point [11]. Assuming dynamic receive focusing in the ultrasound imaging; this fact is valid for each imaging point. Therefore, high spatial frequency components are thrown away to obtain a good viewing image. This filtering process can simply be performed by smoothing the array signals through summation or a smooth weighting and then summation, as it is done in the conventional beamforming methods.

The aforesaid fact can be utilized in the MV beamforming to reduce its computational complexity. Assume that the array signals are filtered by a spatial frequency low pass filter to obtain a low pass array signal. Considering this filter as a decimation filter, the filtered array signal can be down-sampled or decimated in space with no significant aliasing distortion which means the filtered data can be recovered from its decimated version. Therefore, the minimum variance beamforming can be applied on the decimated-filtered data instead of the full data of the primary array, while the performance is almost preserved.

It is worth to note that the transmit focusing makes the desired signal be a narrow band in the spatial frequency domain. This feature permits a high value for the down sampling rate, even with no decimation filter. This is in agreement with the results reported by Nilsen et al. [7] where the beamspace adaptive beamforming through using 3 beams have shown an acceptable performance.

Consider an array of N equally spaced elements, and assume that $\vec{x}[k]$ is the primary array signal at time k after applying delays for dynamic receive focusing and $\vec{f}$ is the tap weight vector of a J-length spatial FIR filter used for smoothing the signal ($J < N$). The array signal after filtering is the convolution of the filter tap and the array signal $\vec{x}[k]$ as follows:

$$\vec{x}_f[k] = F\vec{x}[k] \tag{1}$$

where F is the convolution matrix for vector $\vec{f}$ and $\vec{x}_f[k]$ a $M = N - J + 1$ length vector, is the filtered array signal. Now, assume that $\vec{x}_f[k]$ is decimated to a N_d-length vector called $\vec{x}_d[k]$, such that $\vec{x}_f[k]$ and $\vec{x}_d[k]$ are related through the following equation:

$$x_{d,m}[k] = x_{f,(m-1)r+1}[k] \tag{2}$$

where the second subscript represents the element number and $r = \frac{M-1}{N_d-1}$ is the decimation factor, which is assumed to be an integer. The element-wise relation in (2) can be expressed in a matrix form as:

$$\vec{x}_d[k] = I_d\vec{x}_f[k] \tag{3}$$

where I_d is a $N_d \times M$ matrix obtained through decimation on column of a $M \times M$ identity matrix:

$$[I_d]_{n,m} = \begin{cases} 1, & m = (n-1)r + 1 \\ 0, & otherwise \end{cases} \tag{4}$$

The beamforming process is applied on the decimated data:

$$y[k] = \vec{w}_d^H[k]\vec{x}_d[k] \tag{5}$$

where $\vec{w}_d$ is the apodization vector and y is the beamformer output. In the minimum variance beamforming approach, the apodization vector is calculated to minimize the mean power of output which is obtained through the following optimization problem:

$$\begin{aligned} &Min\left\{E[|y[k]|^2] = \vec{w}_d^H R_d \vec{w}_d\right\} \\ &subject\,to: \ \vec{w}_d^H\vec{1} = 1 \end{aligned} \tag{6}$$

where the steering vector $\vec{1}$ is a $N_d \times 1$ vector of one, assuming dynamic receive focusing, and R_d is the covariance matrix of the decimated data vector:

$$R_d = E\left[\vec{x}_d\vec{x}_d^H\right] \tag{7}$$

The optimal vector $\vec{w}_d$ can be expressed as follows:

$$\vec{w}_d = \frac{R_d^{-1}\vec{1}}{\vec{1}^T R_d^{-1}\vec{1}} \tag{8}$$

It is seen that the decimated MV is an algorithm with $\mathcal{O}(N_d^3)$ computational complexity for the matrix inversion, instead of $\mathcal{O}(N^3)$ in the standard MV.

It is worth to note that if the main sources of the interference signals are assumed those near to the point of interest, the weights on the decimated array can be calculated such that the output power of the primary array is a scaled version of that of the decimated array (see Appendix A). As a result, the minimum power of MV beamformer on the primary array can be reached through applying optimal weights in (8) on the decimated one. Hence, using MV on the decimated array will obtain an image similar to that obtained by the primary full array.

To get a robust response, the MV beamformer usually uses a diagonally load covariance matrix. This technique can be utilized in the DMV and the covariance matrix R_d can diagonally be loaded by a factor α as follows:

$$R_d = R_d + \frac{\alpha}{N_d} trace(R_d) I \tag{9}$$

3. The spatial and temporal averaging

In MV beamformer, the usual way to estimate the covariance matrix through using the single snapshot data is the subarray averaging method. In this method, estimation is done by dividing the aperture into overlapping subarrays and averaging the covariance matrices of each subarray. For the decimated MV, the subarrays are built on the filtered array and then each subarray is decimated into N_d elements, where averaging is done over these decimated subarrays. Assuming each subarray has length L and M is the length of the filtered array, $P = M - L + 1$ subarrays can be used in averaging. Also, consider $\vec{x}_p$ as a $N_d \times 1$ vector containing the decimated data of p'th subarray. Then the decimated covariance matrix is estimated as:

$$\widehat{R}_d[k] = \frac{1}{P}\sum_{p=1}^{P}\vec{x}_p[k]\vec{x}_p^H[k] \tag{10}$$

This technique for estimating the covariance matrix can be considered as the decimation in columns and rows of the covariance matrix of the full array data (see Appendix A). In a similar way, the covariance matrix can be estimated through the temporal and spatial averaging: