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Propagation of shear elastic and electromagnetic waves in one dimensional piezoelectric and piezomagnetic composites

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ABSTRACT

Coupled shear (SH) elastic and electromagnetic (EM) waves propagating oblique to a one dimensional periodic piezoelectric and piezomagnetic composite are investigated using the transfer matrix method. Closed-form expression of the dispersion relations is derived. We find that the band structures of the periodic composite show simultaneously the features of phononic and photonic crystals. Strong interaction between the elastic and EM waves near the center of the Brillouin zone (i.e., phonon-polariton) is revealed. It is shown the elastic branch of the band structures is more sensitive to the piezoelectric effect while the phonon-polariton is more sensitive to the piezomagnetic effect of the composite.

1. Introduction

Phononic crystals are periodic elastic composites [1,2]. The band structures of an elastic wave prorogating in phononic crystals are similar to those of an electromagnetic (EM) wave in photonic crystals [3,4], showing the feature of pass and stop bands. Within stop bands, waves cannot propagate. Such band structures facilitate various applications of phononic crystals, e.g., wave guide, sonic filters, vibration suppression, sound isolation, resonant cavities, acoustic cloak and hyperlens, etc. [5–11] and have promoted intensive interests from scientists and engineers [1–2,5–21].

Piezoelectric materials can generate electric charge upon mechanical loading through piezoelectric effect and vice versa [22]. Recently, periodic piezoelectric composites and their acoustic band structures have been investigated in a number of studies [23–29]. It is found that, in general, the piezoelectric effect can be applied to alter the band gap. In addition, the effects of initial stress have also been investigated [23,29]. In most these studies, the electric field is assumed to be quasi-static. As a result, there is no electromagnetic (EM) wave and only elastic wave exists in these composites. In fact, it is natural to consider the electric field to be dynamic and, consequently, piezoelectric composites are expected to have a dual role as a phononic crystal and a photonic crystal, as that of optomechanical crystals [30]. So far, limited studies on acoustic and EM waves in periodic piezoelectric composites are available [31–38]. Piliposian and his coworkers [31,32] investigated the propagation of coupled SH elastic and EM waves in a one dimensional periodic piezoelectric composite. The full system of Maxwell's equations was considered. It is noted that the crossing of the dispersions of photons and phonons in the long-wavelength limit may indicate a strong coupling of EM wave and lattice vibration, that is, phonon-polariton in periodic piezoelectric composites [33–36]. The problem studied by Piliposian and Ghazaryan [31] was re-visited by Xu et al. [37]. It was found that phonon-polariton occured not only near the center of the Brillouin zone (in the long-wavelength limit) at acoustic frequencies but also in the whole Brillouin zone at optical frequencies, which was confirmed by Ref. [38]. This feature of phonon-polariton makes it possible to manipulate light in a specific path, and new opto-acoustic devices might be designed such as novel laser geometries, optical detection and nondestructive evaluation [31,33].

The piezoelectric composites mentioned above consist of a piezoelectric component and either a piezoelectric or an elastic composite. It is now known that piezoelectric and piezomagnetic composites can have novel magneto-electric effect (ME effect), as that of multiferroic materials [39]. There are several studies on the band structures of two-dimensional magnetoelectroelastic composites, with the electric and magnetic fields approximated as quasi-static [40,41]. In this paper, the band structures of coupled SH elastic and EM waves oblique to a one dimensional periodic composite consisting of piezoelectric and piezomagnetic constituents are explored. Dynamic electric and magnetic fields are taken into account. Using the transfer matrix method, closed form

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solution of the dispersion equation for the coupled wave propagation is obtained. The focus is on the individual role of the piezoelectric and piezomagnetic effects in the band gap and phonon polariton of the composite.

2. Problem description

Propagation of elastic and EM waves in a one dimensional (1D) periodic piezoelectric and piezomagnetic structure is considered in this paper (see, Fig. 1), where a global coordinate system (x, y, and z) is introduced and subdomains-1 and 2 refer to the piezoelectric and piezomagnetic materials, respectively. The elastic and EM waves comply with Maxwell equations of electrodynamics and Newton's equation of equilibrium, that is

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\mathbf{u}} \tag{1}$$

$$\begin{aligned} &\nabla \times \mathbf{E} = -\dot{\mathbf{B}}, \ \nabla \times \mathbf{H} = \dot{\mathbf{D}} \\ &\nabla \cdot \mathbf{D} = 0, \ \nabla \cdot \mathbf{B} = 0 \end{aligned} \tag{2}$$

where over-dot denotes derivative with respect to time t, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{u}$ the elastic displacement vector, ρ the mass density, $\mathbf{E}$ the electric field, $\mathbf{D}$ the electric displacement vector, $\mathbf{B}$ the magnetic induction, and $\mathbf{H}$ is the magnetic field.

The constitutive equations of piezoelectric materials can be expressed as

$$\sigma_{ij} = c_{ijkl}S_{kl} - e_{ijk}E_k \tag{3}$$

$$D_i = e_{ijk}S_{jk} + \varepsilon_{ij}E_j, B_i = \mu_{ij}H_j \tag{4}$$

where $S_{ij} = (u_{i,j} + u_{j,i})/2$ is the infinitesimal strain, and the coefficients (c_{ijkl}, e_{ijk}, ε_{ij}, μ_{ij}) are the elastic, piezoelectric, dielectric, and magnetic permeability coefficients, respectively. Subscripts (i, j, k, l) denote the local coordinates 1, 2, and 3 and summation convention over repeated subscript indices applies.

For the piezomagnetic materials, the constitutive equations have the form of

$$\sigma_{ij} = c_{ijkl}S_{kl} - f_{ijk}H_k \tag{5}$$

$$B_i = f_{ijk}S_{jk} + \mu_{ij}H_j, D_i = \varepsilon_{ij}E_j \tag{6}$$

where f_{ijk} is the piezomagnetic constant. It should be noted that the local crystallographic directions of the piezoelectric material in Fig. 1 is assumed to be consistent with the global coordinate system (i.e., [1 0 0], [0 1 0] and [0 0 1] along the x, y, and z directions, respectively). For the piezomagnetic material, however, its local crystallographic direction [1 0 0] is assumed to be parallel to z, with [0 1 0] and [0 0 1] along the y and x directions, respectively. In doing so, planar wave propagation in the 1D composite given in Fig. 1 can be achieved. Furthermore, only SH wave is considered here, with the nonzero components of displacement, electric field, and magnetic field being u_z, E_x E_y, and H_z, respectively. As a result, the constitutive Eqs. (3)–(6) can be expressed in the global coordinate system as

$$\begin{aligned} \tau_{zx} &= c_{44}S_{zx} - e_{15}E_x \\ \tau_{zy} &= c_{44}S_{zy} - e_{15}E_y \end{aligned} \tag{7}$$

$$\begin{aligned} D_x &= \varepsilon_{11}E_x + e_{15}S_{zx} \\ D_y &= \varepsilon_{11}E_y + e_{15}S_{zy} \\ B_z &= \mu_{33}H_z \end{aligned} \tag{8}$$

for the piezoelectric material having 6 mm symmetry and polarization along the z direction and

$$\begin{aligned} \tau_{zx} &= c_{44}S_{zx} - f_{15}H_z \\ \tau_{zy} &= c_{66}S_{zy} \end{aligned} \tag{9}$$

$$\begin{aligned} &D_x = \varepsilon_{33}E_x, \ D_y = \varepsilon_{11}E_y \\ &B_z = f_{15}S_{zx} + \mu_{11}H_z \end{aligned} \tag{10}$$

for the piezomagnetic material with 6 mm symmetry and polarization in the x direction.

With Eqs. (1)–(10), the dynamic equations of the elastic and EM wave propagating in the x–y plane are given by

$$\begin{aligned} &\tilde{c}_{44}^{(1)} \frac{\partial^2 u_z^{(1)}}{\partial x^2} + \tilde{c}_{44}^{(1)} \frac{\partial^2 u_z^{(1)}}{\partial y^2} = \rho^{(1)} \frac{\partial^2 u_z^{(1)}}{\partial t^2} \\ &\frac{\partial^2 H_z^{(1)}}{\partial x^2} + \frac{\partial^2 H_z^{(1)}}{\partial y^2} = \mu_{33}^{(1)} \varepsilon_{11}^{(1)} \frac{\partial^2 H_z^{(1)}}{\partial t^2} \end{aligned} \tag{11}$$

$$\begin{aligned} &c_{44}^{(2)} \frac{\partial^2 u_z^{(2)}}{\partial x^2} - f_{15} \frac{\partial H_z^{(2)}}{\partial x} + c_{66}^{(2)} \frac{\partial^2 u_z^{(2)}}{\partial y^2} = \rho^{(2)} \frac{\partial^2 u_z^{(2)}}{\partial t^2} \\ &\frac{1}{\varepsilon_{11}^{(2)}} \frac{\partial^2 H_z^{(2)}}{\partial x^2} + \frac{1}{\varepsilon_{33}^{(2)}} \frac{\partial^2 H_z^{(2)}}{\partial y^2} = \mu_{11}^{(2)} \frac{\partial^2 H_z^{(2)}}{\partial t^2} + f_{15} \frac{\partial^2}{\partial t^2} \left(\frac{\partial u_z^{(2)}}{\partial x} \right) \end{aligned} \tag{12}$$

where $\tilde{c}_{44}^{(1)} = c_{44}^{(1)} + e_{15}^2/\varepsilon_{11}^{(1)}$, and superscripts i ($i = 1, 2$) in bracket refer to the subdomains-1 and 2, respectively. Eqs. (11) and (12) are the governing equations of the elastic and EM waves in the piezoelectric and piezomagnetic materials, respectively.

Plane wave solutions of Eqs. (11) and (12) can be assumed in the following forms of

$$\begin{aligned} u_z &= u_{0(x)}^{(j)} e^{i(py - \omega t)} \\ H_z &= H_{0(x)}^{(j)} e^{i(py - \omega t)} \end{aligned}, \ (j = 1, 2) \tag{13}$$

where $i = \sqrt{-1}$, p is a component of the wave vector parallel to the y axis and ω is the circular frequency. In addition, the electric field in the piezoelectric and piezomagnetic materials can be found to be respectively related to the displacement and magnetic field by

$$\begin{aligned} E_x &= \frac{i}{\omega \varepsilon_{11}} \frac{\partial H_z}{\partial y} - \frac{e_{15}}{\varepsilon_{11}} \frac{\partial u_z}{\partial x} \\ E_y &= -\frac{i}{\omega \varepsilon_{11}} \frac{\partial H_z}{\partial x} - \frac{e_{15}}{\varepsilon_{11}} \frac{\partial u_z}{\partial y} \end{aligned} \tag{14}$$

and

$$\begin{aligned} E_x &= \frac{i}{\omega \varepsilon_{11}} \frac{\partial H_z}{\partial y} \\ E_y &= -\frac{i}{\omega \varepsilon_{11}} \frac{\partial H_z}{\partial x} \end{aligned} \tag{15}$$

Substituting Eq. (13) into Eqs. (11) and (12), we obtain the following equations

$$\begin{aligned} &\tilde{c}_{44}^{(1)} \frac{\partial^2 u_{0(x)}^{(1)}}{\partial x^2} - p^2 \tilde{c}_{44}^{(1)} u_{0(x)}^{(1)} = -\omega^2 \rho^{(1)} u_{0(x)}^{(1)} \\ &\frac{\partial^2 H_{0(x)}^{(1)}}{\partial x^2} - p^2 H_{0(x)}^{(1)} = -\omega^2 \mu_{33}^{(1)} \varepsilon_{11}^{(1)} H_{0(x)}^{(1)} \end{aligned} \tag{16}$$

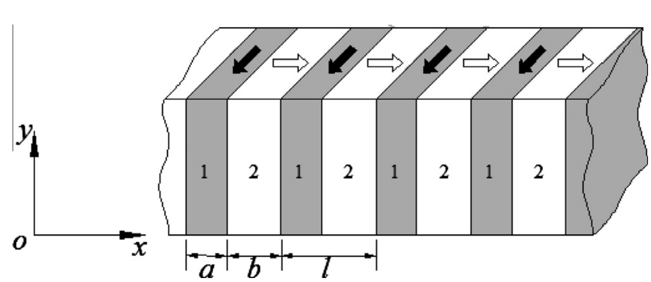

Fig. 1. Schematic of a one dimensional periodic piezoelectric and piezomagnetic composite. Subdomain-1 is a piezoelectric material and subdomain-2 is a piezomagnetic material.