Ultrasonics 55 (2015) 113–122

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

# Modeling of three-dimensional Lamb wave propagation excited by laser pulses

Wenyang Liu [a], Jung-Wuk Hong [b,*]

[a] Third Wave Systems, Minneapolis, United States
[b] Korea Advanced Institute of Science and Technology, Daejeon, Republic of Korea

ARTICLE INFO

*Article history:*
Received 13 January 2014
Received in revised form 6 July 2014
Accepted 10 July 2014
Available online 18 July 2014

*Keywords:*
Finite element methods
Lasers
Simulations
Lamb waves
Wavelet transforms

ABSTRACT

As a type of broadband source of ultrasonic guided waves, laser pulses can be used to launch all modes of interests. In this paper, Lamb waves are excited by imposing heat flux mimicking the supply of the heat from laser pulses, and effects by defects on the received Lamb waves in a plate are investigated by means of the finite element method. In order to alleviate the heavy computational cost in solving the coupled finite element equations, a sub-regioning scheme is employed, and it reduces the computational cost significantly. A comparison of Lamb waves generated by unfocused and line-focused laser sources is conducted. To validate numerical simulations, the group velocity of $A_0$ mode is calculated based on the received signal by using the wavelet transform. The result of $A_0$ mode group velocity is compared with the solution of Rayleigh–Lamb equations, and close agreement is observed. Lamb waves in a plate with defects of different lengths are examined next. The out-of-plane displacement in the plate with a defect is compared with the displacement in the plate without defects, and the wavelet transform is used to determine the arrival times of Lamb waves traveling at the $A_0$ mode group velocity. A strong correlation is observed between the extent of defects and the magnitude of wavelet coefficients.



## 1. Introduction

As the awareness of importance to assess the performance of in-service structures [1] and conduct condition-based maintenance [2] rises, the need for the structural health monitoring and damage detection techniques increases. While the global health monitoring is important to determine whether damages are presented in infrastructures, non-destructive evaluation methods are employed to locate and determine the extent of damages [3]. Elastic and acoustic waves have been widely used for non-destructive evaluations by using, for example, acoustic emission, ultrasonics, and acousto-ultrasonics [4]. Guided by the boundaries of media, Lamb waves are elastic waves propagating in solid plate-like structures. Guided Lamb waves can travel to larger area with only inconsiderable amount of loss in amplitude [5] than conventional pressure waves. Furthermore, received Lamb waves contain rich information not only over the thickness of the plate but also along the travel path. Therefore, Lamb waves are of great potential for structural health monitoring (SHM) purposes.

Progress in understanding Lamb waves has been accompanied by various applications of Lamb waves for damage detection in metallic and composite structures. Tua et al. [6] proposed a method for locating and determining linear cracks using piezo-actuated Lamb waves. Yu and Giurgiutiu [7] presented a method for damage detection using ultrasonic phased arrays. Raghavan and Cesnik [8] proposed analytical solutions for the guided wave fields excited by piezoelectric wafter transducers. Guo and Cawley [9,10] investigated the possibility of using $S_0$ mode for the detection of lamination. Pierce et al. [11] studied the interaction of Lamb waves with various defects such as holes and damage in carbon fiber composites. Moulin et al. [12] proposed a modeling technique to determine the amplitude of Lamb wave modes excited in a composite plate. Apart from defect detection, Lamb waves can also be utilized to estimate elastic properties and sample thicknesses [13]. For example, Dewhurst et al. [14] analyzed the waveform to estimate the thicknesses of metal sheets by considering the sheet velocity and the phase velocity of the antisymmetric mode.

A variety of sources can be used to excite Lamb waves for the non-destructive evaluation purpose. Piezoelectric transducers are a typical source to generate Lamb waves. Giurgiutiu [15] demonstrated the capability of piezoelectric wafer active sensors to excite and detect tuned Lamb waves. Moulin et al. [16] embedded piezoelectric transducers inside a composite plate to effectively generate Lamb waves. Compared to piezoelectric transducers that require direct physical contact to structural surfaces, non-contact

* Corresponding author.
*E-mail addresses:* jwhong@alum.mit.edu, j.hong@kaist.ac.kr (J.-W. Hong).

http://dx.doi.org/10.1016/j.ultras.2014.07.006
0041-624X/

techniques have their unique attractiveness for the generation and detection of Lamb waves. Air-coupled ultrasonic inspection is ideal for materials that would be easily damaged by physical contact [17]. Laser pulses are conveniently used as a broadband source to launch all modes of interests [18].

Hutchins and Lundgren [13] described a method to generate and detect Lamb waves in thin materials using laser techniques. Staszewski et al. [4] used a laser Doppler velocimeter as a non-contact method to detect low-frequency Lamb waves. As a result of absorption of laser energy, thermoelastic deformations are induced in the specimen. Thermomechanical problems have been studied for a long time, and the finite element method has been used to solve the thermomechanical coupled formulations. For example, Armero and Simo [19] proposed a method for non-linear coupled thermomechanical problems. Tian et al. [20] studied finite element equations of the generalized thermoelasticity in which two relaxation times are considered. Serra and Bonaldi [21] utilized the finite element method for the thermoelastic damping analysis.

With information gathered for non-destructive evaluations, analyzing the received data is a critical step to determine structural damages. The wavelet transform is a data analysis tool that decomposes a signal into time–frequency representations [22]. Hayashi et al. [23] used the wavelet transform of Lamb waves to estimate the thickness and elastic properties of metallic foils. Choi and Hong [24] investigated wavelet coefficients of ultrasonic signals quantitatively. White et al. [25] presented an analytical solution for wavelet transformed ultrasonic signals and applied the derived equation to calculate times of flight of multiple echoes. An approach to determine the fracture source location using the wavelet transform was proposed by Jeong and Jang [26]. Niethammer et al. [27] compared four time–frequency representations including the reassigned spectrogram, the reassigned scalogram, the smoothed Wigner–Ville distribution, and the Hilbert spectrum. Clough and Edwards [28] investigated surface wave enhancements for Lamb waves propagating in plates and observed frequency enhancement in the near field of surface defects. Burrows et al. [29] used the finite element method to study the propagation of Lamb waves generated by a pulsed laser beam and interaction with defects. The thermomechanical simulations in [29] are performed with two-dimensional models.

In this paper, Lamb waves excited by simulated laser pulses are studied, and effects of defects on the received Lamb waves at a monitoring location are investigated. The three-dimensional thermal–mechanical coupling analysis using the finite element method is performed. To reduce the computational cost involved in solving thermomechanical coupled formulations, a sub-regioning scheme that divides the plate into mechanical and thermomechanical subregions is employed. A comparison of generated Lamb waves is conducted for unfocused and line-focused laser sources. To validate numerical simulations, the group velocity of $A_0$ mode is calculated based on the detected signal using the wavelet transform, and the result is then compared with the solution of Rayleigh–Lamb equations. Different from the study in [28] in which the defect depth is the factor that has been examined, defects of different lengths in a plate and their effects on the wavelet coefficients are studied in this paper. The out-of-plane displacement in the plate with a defect is compared to the displacement in the plate without defects, and the wavelet transform is used to determine the arrival times of the $A_0$ mode and the extent of defects in the plate.

## 2. Theory

### 2.1. Finite element formulation for thermoelastic analysis

In the following, we summarize the thermomechanical formulation in Ref [30]. The finite element method has been applied to analyze a wide range of engineering problems [31]. To model Lamb waves excited by laser pulses, the coupled thermoelasticity is considered. The equation of motion in local form [31] and the local balance equation of entropy [21] are written, respectively, as

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i, \tag{1}$$

$$T_0 \dot{s} = -q_{i,i}, \tag{2}$$

where $\sigma_{ij}$ are stresses, $f_i$ are body forces, $\rho$ is the density, and $u_i$ are displacements, $T_0$ is the equilibrium temperature, $\dot{s}$ is the entropy rate per unit volume, and $q_i$ is the thermal flux. For a well-posed boundary value problem, both mechanical and thermal boundary conditions must be satisfied. The boundary $\Gamma$ of a domain can be decomposed as $\Gamma = \Gamma_\sigma \cup \Gamma_u = \Gamma_Q \cup \Gamma_\theta$ [19]. The traction boundary condition $\Gamma_\sigma$ and the displacement boundary condition $\Gamma_u$ are given as

$$\sigma_{ij} n_j = \bar{\sigma}_i \quad \text{on } \Gamma_\sigma, \tag{3}$$

$$u_i = \bar{u}_i \quad \text{on } \Gamma_u, \tag{4}$$

where $\bar{\sigma}_i$ are the prescribed surface tractions, and $\bar{u}_i$ are the prescribed displacements. The heat flux boundary $\Gamma_Q$ and the temperature boundary $\Gamma_\theta$ are specified as

$$q_i n_i = \overline{Q} \quad \text{on } \Gamma_Q, \tag{5}$$

$$\theta = \bar{\theta} \quad \text{on } \Gamma_\theta, \tag{6}$$

where $\overline{Q}$ and $\bar{\theta}$ are the prescribed heat flux and temperature specified at thermal boundaries. The constitutive equations are stated as

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} - \beta_{ij} \theta, \tag{7}$$

$$s = \beta_{ij} \varepsilon_{ij} + \frac{c_E}{T_0} \theta, \tag{8}$$

where $C_{ijkl}$ is the elasticity tensor, $\varepsilon_{ij}$ is the strain tensor, $c_E$ is the specific heat capacity, and $\beta_{ij}$ is the thermomechanical coupling tensor which is of the form for isotropic materials as [19]

$$\boldsymbol{\beta} = \begin{bmatrix} 3\kappa\alpha & 0 & 0 \\ 0 & 3\kappa\alpha & 0 \\ 0 & 0 & 3\kappa\alpha \end{bmatrix}, \tag{9}$$

where $\kappa$ and $\alpha$ denote the bulk modulus and the coefficient of thermal expansion, respectively. The strain–displacement relation is expressed as $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, and Fourier's heat conduction law, which states the relation between the heat flux and the temperature gradient, is given by

$$q_i = -k_{ij} \theta_{,j}, \tag{10}$$

where $k_{ij}$ is the thermal conductivity tensor as

$$\mathbf{k} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}. \tag{11}$$

Applying the principle of virtual work and the principle of virtual temperature to Eqs. (1) and (2), respectively, we have

$$\int_V (\sigma_{ij,j} + f_i - \rho \ddot{u}_i) \delta u_i \, dV = 0, \tag{12}$$

$$\int_V (T_0 \dot{s} + q_{i,i}) \delta\theta \, dV = 0. \tag{13}$$

After integrating by parts and applying the divergence theorem, the weak formulations are obtained as

$$-\int_V \sigma_{ij} \delta\varepsilon_{ij} \, dV + \int_\Gamma \sigma_{ij} n_j \delta u_i \, dA + \int_V (f_i - \rho \ddot{u}_i) \delta u_i \, dV = 0, \tag{14}$$

$$\int_V T_0 \dot{s} \delta\theta \, dV - \int_V q_i \delta\theta_{,i} \, dV + \int_\Gamma q_i n_i \delta\theta \, dA = 0. \tag{15}$$

Substituting Eqs. (3)–(10) into Eqs. (14) and (15), we then arrive at the equations as