Application of orthogonality-relation for the separation of Lamb modes at a plate edge: Numerical and experimental predictions

M. Ratassepp [a,*], A. Klauson [a], F. Chati [b], F. Léon [b], D. Décultot [b], G. Maze [b], M. Fritzsche [c]

[a] *Department of Mechanics, Tallinn University of Technology, Ehitajate tee 5, Tallinn 19086, Estonia*
[b] *Laboratoire Ondes et Milieux Complexes UMR CNRS 6294, Université Le Havre, 75 rue Bellot CS 50540, Le Havre, France*
[c] *Polytec GmbH, Sales International, Waldbronn, Germany*

ABSTRACT

In this study the orthogonality relation-based method for post-processing finite element (FE) predictions and experimental measurements is applied in order to separate Lamb modes at a plate edge at normal incidence. The scattered wave field from the free edge is assumed to be a superposition of all the eigenmodes of an infinite plate. The eigenmode amplitudes of the reflected wave field are determined by implementing the orthogonality-based method on the measured plate edge displacements. Overlapping wavepackets of Lamb modes at a plate edge are simulated by using the FE model and the experiment in the case of an incident S_0 mode in a plate with a notch. In the experiment a 3D Scanning Laser Doppler Vibrometer (3D SLDV) (Johansmann and Sauer, 2005) is used to measure 3 dimensional vibrations and thus the edge two-dimensional displacement components simultaneously. It is demonstrated that it is possible to extract signals of various propagating and non-propagating modes in time-domain. The influences of the errors in practical measurements on the extraction procedure have also been studied.

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1. Introduction

Lamb modes are widely used for non-destructive evaluation of plate-like structures. One of the principal difficulties to a robust, rapid and accurate defect localization technique based on wide-band, multi-mode guided waves is the multimodal response signal [2–6]. In general it is not possible to avoid multimodality in Lamb wave testing. Even if the incident wave is a pure Lamb mode, the interaction of a wave with a defect or structural feature can result in a complicated multimode signal due to mode conversions, since there may exist at least two propagating modes in a plate at any chosen testing frequency. It is important to separate wave-packets of different modes, as the relative amplitude of each received mode is dependent on the geometry and location of the defects. This provides more information about the defect and is vital for the development of inverse procedures for the defect characterization [7–9].

In general, the existing mode separation techniques make use of the wave field data measured on the plate surface. The most well-known is the classical two-dimensional spatial fast Fourier transform (FFT) technique that uses time records from a series of equally spaced points along a plate [2]. The result is the three-dimensional wave field amplitude spectrum where the individual Lamb modes

can be resolved and their amplitudes measured. The mode decomposition can be done from nonstationary signals even from a single measurement by using more advanced signal processing techniques such as reassigned spectrogram [4] or dispersion compensation method [5]. Recently, a group velocity ratio and mode amplitude ratio rules have been proposed to separate S_0 and A_0 mode [6]. However, there are cases in practice when the plate surface is inaccessible for testing (covered by isolation or other layers in multilayered plates) and the only location for monitoring is the plate edge. In this case an attractive approach to separate the modes is to use the Lamb mode orthogonality-relation [10]. In comparison to the classical spatial FFT, the orthogonality-relation based method allows the reduction of the number of monitoring points and there is no need for special mode filtering procedures as obtaining amplitudes of the modes is straight-forward. Although this method requires the measurement of the through-thickness displacements and also the stress field components, the orthogonality-relation at the plate edge is simplified as the stresses equal to zero. Therefore only two displacement components have to be measured at a plate edge which can make the method possible for practical use.

The orthogonality of the Lamb modes was shown already a long time ago [11] and this property has helped to solve several wave propagation and scattering problems in structures. The idea of using orthogonality to extract individual Lamb modes from the scattered wave fields is not new. Moreau et al. [10] used the

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* Corresponding author.
  E-mail address: madis.ratassepp@ttu.ee (M. Ratassepp).

orthogonality relation to calculate the reflection and transmission coefficient of isolated modes in case of a pure Lamb mode incident on a notch-like defect. In another paper [12] they also showed that the proposed technique can be extended on three-dimensional guided wave scattering problems in plates. The use of modal decomposition method with the orthogonality relation has allowed solving the Lamb wave interaction with a plate edge [13] and delaminated plate [14]. Gunawan and Hirose [15] derived a generalized orthogonality relationship for the Lamb modes of oblique scattering on the free edge of a plate. They used it to develop a mode decomposition technique for an elastodynamic field and semi-analytically obtained the reflection coefficients for the oblique incidence problem.

In addition, it is important to understand the interaction of Lamb modes with a plate edge which has been studied and reported quite extensively [16–23]. It has been shown that the incident wave interacting with a free plate edge gives rise to a system of reflected waves, consisting of propagating and non-propagating modes. Above the cut-off frequencies of higher order modes the energy carried by the incident mode can be distributed among the other possible reflected propagating modes [13,16–18,22]. At certain frequencies the edge motion is dominated by the movement of non-propagating modes which hinders the measurement of propagating modes [19,20]. All these effects can be investigated with the proposed approach.

In this paper, we develop a numerical and experimental procedure based on the Lamb mode orthogonality to separate various modes at a plate edge in a plane strain condition. Overlapping wavepackets of S_0 and A_0 Lamb modes arriving at a plate edge are simulated by using the FE model and generated in the experiment of an incident S_0 mode in a plate with a notch. The required wave field displacement components for the experimental procedure are measured with a 3D SLDV. It is demonstrated that it is possible to extract the signals of several propagating and non-propagating modes in time-domain.

2. Orthogonality-relation of Lamb-modes at a plate edge

Fig. 1 shows the two-dimensional Lamb mode propagating towards the edge of a semi-infinite plate along the x-direction. The plate medium is considered to be isotropic, homogeneous and elastic; plane strain conditions are considered.

The displacements and stresses of each Lamb mode with order n are expressed in the vector form:

$$\boldsymbol{u}_n = \begin{pmatrix} u_x \\ u_y \end{pmatrix}_n e^{i(\omega t - k_n x)}, \tag{1}$$

$$\boldsymbol{\sigma}_n = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix}_n e^{i(\omega t - k_n x)}, \tag{2}$$
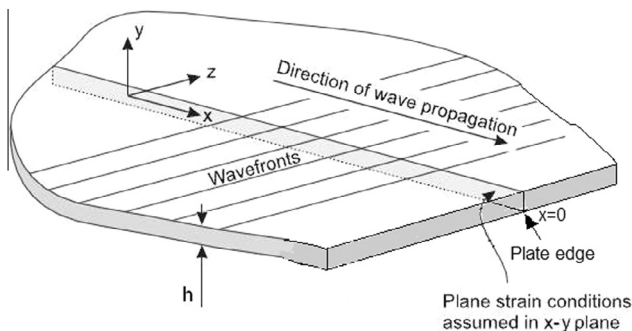

Fig. 1. Geometry of the problem.

where u_x, u_y, σ_{xx}, σ_{xy} are displacement and stress profiles in y-coordinate respectively, t is time, ω is angular frequency and k_n is the complex wave number of mode n. For a detailed description, the reader should look in [24].

The total wave field can be written as a modal series of Lamb eigenmodes which must satisfy stress-free boundary conditions on the free edge $x = 0$:

$$\begin{pmatrix} S_{xx} \\ S_{xy} \end{pmatrix} = \sum_m r_{m,\pm} \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix}_m = 0, \quad \begin{pmatrix} U_x \\ U_y \end{pmatrix} = \sum_m r_{m,\pm} \begin{pmatrix} u_x \\ u_y \end{pmatrix}_m \tag{3}$$

where $r_{m,\pm}$ is the complex reflection amplitude of mode $m_\pm$. The + sign marks the mode propagating in positive direction and – sign in negative direction, respectively.

The general orthogonality relation [14], which involves a scalar product between the displacement and stress distributions of two modes m and n is considered at a given position along the plate:

$$\int_0^h \left[(\sigma_{xy})_n (u_y)_m + (\sigma_{xy})_m (u_y)_n - (\sigma_{xx})_n (u_x)_m - (\sigma_{xx})_m (u_x)_n \right] dy$$

$$= a(n)\delta_{mn}, a(n) = 2 \int_0^h \left[(\sigma_{xy})_n (u_y)_n - (\sigma_{xx})_n (u_x)_n \right] dy, \tag{4}$$

where $a(n)$ is the normalization factor and δ_{mn} is the Kronecker delta symbol. One can verify that all pairs of Lamb eigenmodes with different wave numbers are orthogonal using Eq. (4). The modes propagating in positive and negative are also orthogonal, since for this case $k_{n+} = -k_{n-}$ and all terms in Eq. (4) are cancelled.

Applying the orthogonality condition to the total wave field on the free edge, the relation between integral $A(n)$ and reflection amplitude r_n and normalization factor $a(n)$ can be obtained

$$A(n) = \int_0^h \left[(\sigma_{xy})_n U_y + S_{xy}(u_y)_n - (\sigma_{xx})_n U_x - S_{xx}(u_x)_n \right] dy$$

$$= \sum_m r_{m,\pm} \int_0^h \left[(\sigma_{xy})_n (u_y)_m + (\sigma_{xy})_m (u_y)_n - (\sigma_{xx})_n (u_x)_m \right.$$

$$\left. - (\sigma_{xx})_m (u_x)_n \right] dy = \sum_m r_{m,\pm} a(n)\delta_{mn} = r_n a(n). \tag{5}$$

By using boundary conditions (3) and Eq. (4), one obtains the reflection amplitude

$$r_n = \frac{A(n)}{a(n)} = \frac{\int_0^h [(\sigma_{xy})_n U_y - (\sigma_{xx})_n U_x] dy}{2 \int_0^h \left[(\sigma_{xy})_n (u_y)_n - (\sigma_{xx})_n (u_x)_n \right] dy}. \tag{6}$$

3. FE modeling and post-processing of the results

Fig. 2 shows the FE model for Lamb modes scattered by a notch and reflected at a plate edge. Wave propagation is simulated by using the finite element modeling software Ansys [25]. Lamb mode S_0 is generated on the left edge by prescribing identical displacement in x-direction at all nodes on the edge. The excited mode propagates along the plate, interacts with a notch in the plate and reaches the plate edge. The interaction phenomenon causes the scattering of Lamb modes, reflected from and transmitted past the defect. Multiple reflections can take place as the notch is rather close to the plate edge. The total acoustic field in the guide can therefore be very complicated since it results from the superposition of the incident and all the scattered modes: a series of propagating modes, plus an infinite number of non-propagating modes. The time-domain signals of U_x and U_y displacement components are monitored along the edge in y-direction in 9 points with step of 0.375 mm. For the separation of the modes these signals are transformed into frequency-domain by using Fast Fourier transform. As the excitation signal is chosen to be a Hanning-windowed toneburst, the data processing is performed over a given range of