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Numerical investigation of leaky modes in helical structural waveguides embedded into a solid medium

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ABSTRACT

Helical multi-wire cables are widely used in bridges (suspended or prestressed) and anchored retaining wall constructions. Such structures can be damaged or degraded due to corrosion and fatigue. Non destructive evaluation techniques are required to reveal defects inside cable structures. Among these numerous techniques, elastic guided waves are of potential interest owing to their ability to propagate over long distances. However in civil engineering, cables are often buried or grouted in large solid media that can be considered as unbounded. Waves can strongly attenuate along the guide axis due to the energy leakage into the surrounding medium, which reduces the propagating distance. This energy leakage can be enhanced in helical structures, which further complicates their inspection. Searching modes with low attenuation becomes necessary. The goal of this work is to propose a numerical approach to compute modes in embedded helical structures, combining the so-called semi analytical finite element method and a radial perfectly matched layer technique. Two types of radial perfectly matched layer, centered and off-centered, are considered. Both are implemented in a twisting coordinate system which preserves translational invariance. The centered configuration is validated thanks to the twisted cylinder test case. The effect of twist on the eigenspectrum is briefly discussed. Then, an embedded helical wire of circular cross-section is considered. The off-centered configuration is shown to give the same results as the centered one. The effect of twist on modal attenuation is investigated. Finally, computations are performed for a seven-wire strand embedded into concrete, widely used in civil engineering cables.

1. Introduction

Helical structures are present in various domains, such as electromagnetism and civil engineering. A typical example in civil engineering is multi-wire cables, widely used in bridges (suspended or prestressed) and anchored retaining wall constructions. Cables can be damaged or degraded due to corrosion and fatigue. Non destructive evaluation (NDE) techniques are required to evaluate defects inside cable structures. Among these numerous techniques, elastic guided waves are of particular interest owing to their ability to propagate over long distances. Because such waves are multimodal and dispersive, modeling tools are required in practice for interpreting measurements and optimizing inspection techniques.

Due to the complexity of equations in helical systems, analytical solutions are difficult or impossible to achieve. Purely numerical approaches have to be adopted. A classical method that has been widely used for straight waveguides is the so-called semi-analytical finite element (SAFE) method. This method restricts the FE discretization to transverse directions only [1–4]. It has been applied for modeling closed helical waveguides (guides in vacuum) in Ref. [5], where the SAFE modeling of a free single helical wire has been presented based on helical coordinates. A particular twisting coordinate system has then been proposed for the analysis of single helical wires as well as multi-wire strands [6].

Regardless helicity, structural waveguides are often embedded in large solid media that can be considered as unbounded. Waves can radiate energy into the surrounding medium and strongly attenuate along the guide axis, which reduces the propagation distance. Such wave modes are referred to as leaky modes [7,8]. This energy leakage can be enhanced in curved or helical structures, which makes their NDE more difficult. The curvature effect on radiation loss has been thoroughly studied in electromagnetism [9–13] and sometimes investigated in elastodynamics [14,15]. In the context of NDE, searching the less attenuated modes is necessary in order to maximize the inspection distance.

As opposed to closed waveguides, the numerical modeling of embedded waveguides encounters two difficulties: the cross-section is unbounded and the amplitude of leaky modes transversely

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grows [16–19]. In order to overcome these difficulties, the SAFE method must be combined with other numerical techniques.

As far as straight waveguides are concerned, several techniques have been recently proposed to extend the SAFE method to guides embedded in a solid matrix. A simple numerical procedure is the absorbing layer (AL) method proposed in Refs. [20,21], which consists in creating artificial viscoelastic layers in the surrounding medium for absorbing waves. Instead of using artificial layers, Mazzotti et al. [22] have combined the boundary element method (BEM) with the SAFE method, which avoids the discretization of the unbounded surrounding domain. An alternative technique is the perfectly matched layer (PML) method. Recently, the authors have presented and analyzed SAFE-PML methods for modeling embedded solid multi-layer plates [23] and three dimensional waveguides of arbitrary cross-section [24]. These works are yet limited to straight waveguides. In electromagnetism, a SAFE-PML formulation has been proposed for the analysis of twisted microstructured optical fibers [25,26]. Yet to the authors knowledge, the modeling of embedded helical structures has not yet been considered in elastodynamics.

The goal of this paper is to propose a SAFE-PML technique to compute leaky modes in embedded helical structures. The twisted SAFE-PML method is described in Section 2. The equilibrium equations of elastodynamics are written in twisting coordinates to account for the helical geometry. In this coordinate system, a radial PML is applied. This radial PML can be centered or off-centered. The method is validated in Section 3 thanks to the cylindrical bar test case, which can support any arbitrary twist. The effect of twist on the eigenspectrum is briefly discussed. Two numerical applications are then presented in Section 4. The first example consists in studying an embedded helical wire of circular cross-section. The effect of twist on the axial attenuation of modes is investigated. The second application is a seven-wire strand embedded into concrete. Seven-wire strands are widely used in civil engineering cables. They are typically made by one straight cylindrical wire surrounded by one layer of six helical wires.

2. Numerical method

2.1. Elastodynamics in twisting coordinates

Let us introduce a twisting coordinate system (x, y, z) defined from the Cartesian coordinates (X, Y, Z) [6]:

$$\begin{aligned} x &= X\cos(\tau Z) + Y\sin(\tau Z) \\ y &= -X\sin(\tau Z) + Y\cos(\tau Z) \\ z &= Z \end{aligned} \tag{1}$$

where τ denotes the torsion, which characterizes the rotation rate of the (x, y) plane along the z axis. In the Cartesian basis $(\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z)$, the twisting basis $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ is expressed as follows:

$$\begin{aligned} \mathbf{e}_x &= \cos(\tau Z)\mathbf{e}_X + \sin(\tau Z)\mathbf{e}_Y \\ \mathbf{e}_y &= -\sin(\tau Z)\mathbf{e}_X + \cos(\tau Z)\mathbf{e}_Y \\ \mathbf{e}_z &= \mathbf{e}_Z \end{aligned} \tag{2}$$

One considers a three-dimensional helical waveguide $\tilde{\Omega} = \tilde{S} \times]-\infty, +\infty[$ whose cross-section $\tilde{S}$ lies in the transverse $(\tilde{x}, \tilde{y})$ plane and is invariant along the z axis. The tilde notation will be explained through the introduction of PML in Section 2.2.

The time harmonic dependence is chosen as $e^{-i\omega t}$. Linear elastic materials are assumed. As this study focuses on eigenmodes, acoustic sources and external forces are discarded. In the twisting coordinate system, the three-dimensional variational formulation of elastodynamics is given by [6]:

$$\int_{\tilde{\Omega}} \delta\tilde{\boldsymbol{\epsilon}}^T \tilde{\boldsymbol{\sigma}} d\tilde{\Omega} - \omega^2 \int_{\tilde{\Omega}} \tilde{\rho}\, \delta\tilde{\mathbf{u}}^T \tilde{\mathbf{u}} d\tilde{\Omega} = 0 \tag{3}$$

where $d\tilde{\Omega} = d\tilde{x}d\tilde{y}dz$. The strain–displacement relation is:

$$\tilde{\boldsymbol{\epsilon}} = \left(\mathbf{L}_{\tilde{S}} + \mathbf{L}_z \frac{\partial}{\partial z}\right)\tilde{\mathbf{u}} \tag{4}$$

where the operators separating transverse from axial derivatives are:

$$\mathbf{L}_{\tilde{S}} = \begin{bmatrix} \partial/\partial\tilde{x} & 0 & 0 \\ 0 & \partial/\partial\tilde{y} & 0 \\ 0 & 0 & \Lambda_{\tilde{S}} \\ \partial/\partial\tilde{y} & \partial/\partial\tilde{x} & 0 \\ \Lambda_{\tilde{S}} & -\tau & \partial/\partial\tilde{x} \\ \tau & \Lambda_{\tilde{S}} & \partial/\partial\tilde{y} \end{bmatrix}, \quad \mathbf{L}_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{5}$$

with $\Lambda_{\tilde{S}} = \tau\tilde{y}\frac{\partial}{\partial\tilde{x}} - \tau\tilde{x}\frac{\partial}{\partial\tilde{y}}$.

The formulation (3) holds for any kinematically admissible displacement $\delta\tilde{\mathbf{u}} = [\delta\tilde{u}_x \delta\tilde{u}_y \delta\tilde{u}_z]^T$. The superscript T denotes the matrix transpose. The notation $\delta\tilde{\boldsymbol{\epsilon}} = [\delta\tilde{\epsilon}_{xx}\delta\tilde{\epsilon}_{yy}\delta\tilde{\epsilon}_{zz}\; 2\delta\tilde{\epsilon}_{xy} 2\delta\tilde{\epsilon}_{xz} 2\delta\tilde{\epsilon}_{yz}]^T$ is the virtual strain vector. Similarly, $\tilde{\boldsymbol{\sigma}} = [\tilde{\sigma}_{xx}\tilde{\sigma}_{yy}\tilde{\sigma}_{zz}\tilde{\sigma}_{xy}\tilde{\sigma}_{xz}\tilde{\sigma}_{yz}]^T$ denotes the stress vector. Vector components are expressed in the twisting basis $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$. The stress–strain relation is given by $\tilde{\boldsymbol{\sigma}} = \tilde{\mathbf{C}}\tilde{\boldsymbol{\epsilon}}$, where $\tilde{\mathbf{C}}$ is the matrix of material properties. $\tilde{\rho}$ is the material mass density. We assume that $\tilde{\mathbf{C}}$ and $\tilde{\rho}$ depend only on the twisting transverse coordinates $(\tilde{x}, \tilde{y})$, which means that the problem is translationally invariant along the z axis.

2.2. Radial PML

Let us define the cylindrical representation (r, θ, z) of the twisting coordinates (x, y, z) from: $\tilde{x} = x_{O'} + \tilde{r}\cos\theta, \tilde{y} = y_{O'} + \tilde{r}\sin\theta$. In the (x, y) plane, the point O' of coordinates $(x_{O'}, y_{O'})$ is the center of this cylindrical system. $x_{O'}$ and $y_{O'}$ are independent of the axial coordinate z. As shown by Fig. 1a, the point O' thus defines a helix as it travels in the z direction. A helix curve can be characterized by two parameters: R_h, the helix radius in the (x, y) plane and L_h, the helix step along the z axis. The torsion of the twisting coordinate system attached to the helix is defined by $\tau = 2\pi/L_h$ [6]. In the remainder of this paper, we will set $y_{O'} = 0$ without loss of generality.

For clarity, the Jacobian matrix of the transformation from $(\tilde{x}, \tilde{y}, z)$ to $(\tilde{r}, \theta, z)$ and its inverse are given by:

$$\begin{aligned} \mathbf{J} &= \begin{bmatrix} \partial\tilde{x}/\partial\tilde{r} & \partial\tilde{x}/\partial\theta & \partial\tilde{x}/\partial z \\ \partial\tilde{y}/\partial\tilde{r} & \partial\tilde{y}/\partial\theta & \partial\tilde{y}/\partial z \\ \partial z/\partial\tilde{r} & \partial z/\partial\theta & \partial z/\partial z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\tilde{r}\sin\theta & 0 \\ \sin\theta & \tilde{r}\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \mathbf{J}^{-1} &= \begin{bmatrix} \partial\tilde{r}/\partial\tilde{x} & \partial\tilde{r}/\partial\tilde{y} & \partial\tilde{r}/\partial z \\ \partial\theta/\partial\tilde{x} & \partial\theta/\partial\tilde{y} & \partial\theta/\partial z \\ \partial z/\partial\tilde{x} & \partial z/\partial\tilde{y} & \partial z/\partial z \end{bmatrix} = \frac{1}{\tilde{r}} \begin{bmatrix} \tilde{r}\cos\theta & \tilde{r}\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & \tilde{r} \end{bmatrix} \end{aligned} \tag{6}$$

The formulation (3) is now transformed into cylindrical coordinates, but with vectors and tensors still expressed in the basis $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$. One has $d\tilde{\Omega} = \tilde{r}d\tilde{r}d\theta dz$. The operator $\mathbf{L}_z$ of the strain–displacement relation (4) is unchanged. The operator $\mathbf{L}_{\tilde{S}}$ is rewritten, thanks to the Jacobian matrix $\mathbf{J}^{-1}$ in Eq. (6), by replacing $\partial/\partial\tilde{x}, \partial/\partial\tilde{y}$ and $\Lambda_{\tilde{S}}$ with:

$$\begin{aligned} &\frac{\partial}{\partial\tilde{x}} = \cos\theta\frac{\partial}{\partial\tilde{r}} - \frac{\sin\theta}{\tilde{r}}\frac{\partial}{\partial\theta}, \quad \frac{\partial}{\partial\tilde{y}} = \sin\theta\frac{\partial}{\partial\tilde{r}} + \frac{\cos\theta}{\tilde{r}}\frac{\partial}{\partial\theta}, \\ &\Lambda_{\tilde{S}} = -\tau x_{O'}\sin\theta\frac{\partial}{\partial\tilde{r}} - \tau\left(x_{O'}\frac{\cos\theta}{\tilde{r}} + 1\right)\frac{\partial}{\partial\theta}. \end{aligned} \tag{7}$$

The main difficulty for modeling an embedded waveguide is the unbounded nature of its cross-section. The basic idea proposed in this paper consists in closing the cross-section thanks to a PML