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Diffraction of collinear correlated photon pairs by an ultrasonic wave within Raman–Nath and intermediate region

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ABSTRACT

The phenomenon of collinear correlated photon pairs diffraction by an ultrasonic wave is investigated within Raman–Nath and intermediate region. The numbers of single photons and photon pairs counts in discrete diffraction orders were measured as functions of the Raman–Nath parameter. Similarly, the number of coincidence photon counts in separate diffraction orders was also investigated. It was shown experimentally that the phenomenon of photon pairs diffraction by an ultrasonic wave happens at angles identical to those corresponding to single photons diffraction. It was also demonstrated that in case of Raman–Nath diffraction the number of photon pairs in a selected, n^{th}, diffraction order varies with the Raman–Nath parameter changes as an n^{th} order Bessel function of the first kind, raised to the fourth power. Whilst in the so-called intermediate diffraction zone, the number of diffracted photon pairs varies as squared intensity of a diffracted light beam consisting of single photons. Moreover, it was revealed that correlations between photons in selected diffraction orders change with the Raman–Nath parameter variation as products of relevant intensities of light in the considered diffraction orders. Finally, it should be emphasized that the presented formulae describing diffraction of collinear correlated proton pairs by an ultrasonic wave are in a very good agreement with corresponding experimental data, for both Raman–Nath and intermediate diffraction.

1. Introduction

To the author's knowledge, the phenomenon of photon pairs diffraction by an ultrasonic wave has not been yet investigated experimentally, either in the Raman–Nath or intermediate diffraction region. In 2012, L.B. Deng [1] published a theoretical work describing interaction of single photons and entangled photon pairs with an ultrasonic wave. In his study he showed that the probability distribution of degenerate entangled photon pairs in diffraction orders, in case of Raman–Nath diffraction, changes twice faster as a function of the Raman–Nath parameter than it takes place for single photons. Whereas relevant probability distributions corresponding to single photons, derived by Deng, appear to be described by formulas identical to those given by C.V. Raman and N.S. Nagendra Nath in case of a plane light wave diffraction by an ultrasonic beam. Moreover, he revealed that in case of degenerate entangled photon pairs the diffraction phenomenon actually takes place at angles which are twice smaller than relevant angles corresponding to single photons.

The aim of this work was to investigate experimentally both single photons and collinear correlated photon pairs diffraction by an ultrasonic wave within the Raman–Nath and intermediate diffraction regions. A BBO crystal was used to produce single photons and collinear correlated photon pairs via type-I spontaneous parametric down-conversion.

The phenomenon of photon pairs interaction with an ultrasonic wave at the Bragg angle is not discussed in the work as it has already been investigated experimentally for non-collinear photon pairs [2] and, recently, also for collinear ones, in a separate paper by the author [3].

2. Elementary considerations

The most famous theory of the diffraction of light by ultrasonic waves was developed by C.V. Raman and N.S. Nagendra Nath and was written during the period 1935–1936 in the series of five papers [4]. The first three [4, Parts I–III] papers contain the theory which is based on the assumption that the light, during its passage through the ultrasonically disturbed medium, is only affected in the phase. In the two last papers of Raman and Nath [4, Parts IV–V], besides variation of the phase also amplitude changes of the

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light wave are taken into account. At present, the theory which is based on the former assumption is called the elementary Raman–Nath theory [4, Parts I–III], as opposed to the generalized one [4, Parts IV–V,5], which considers both phase and amplitude alteration of light beam during its interaction with an ultrasonic wave. According to the generalized Raman–Nath theory, the amplitudes ϕ_n of ultrasonically diffracted light are solutions to the following system of difference–differential equations [5]:

$$\frac{d\phi_n}{dz} + \frac{v}{2L}(\phi_{n-1} - \phi_{n+1}) = i\frac{nQ}{2L}\left(n + 2\frac{\mu_0\Lambda}{\lambda}\sin\Theta\right)\phi_n, \tag{1}$$

where ($n = -\infty, \ldots +\infty$) and $i = \sqrt{-1}$.

With the boundary condition $\phi_0(0) = 1$ and $\phi_n(0) = 0$, for $n \neq 0$.

The above equation set depends on two parameters, called:

$$\text{Raman–Nath parameter} \quad v = \frac{2\pi\mu_1 L}{\lambda}, \tag{2}$$

$$\text{and Klein–Cook parameter} \quad Q = \frac{2\pi\lambda L}{\mu_0\Lambda^2}, \tag{3}$$

where λ is the wavelength of the (incident) light wave in a vacuum, Λ denotes the wavelength of the ultrasonic wave in the considered medium, μ_0 stands for the refractive index of the (undisturbed) medium in which the ultrasonic wave propagates, z represents the propagation direction of the incident light wave, μ_1 is the maximum variation of the refractive index of the medium and is proportional to the sound pressure amplitude.

The Raman–Nath equations set (1) was derived for a plane light wave of the angular frequency ω and wavenumber $k = 2\pi/\lambda$, incidenting at an angle Θ an ultrasonic beam of the width L. The ultrasonic wave induces periodic temporal and spatial variations of the refractive index of the medium, which can be expressed as:

$$\mu(x,t) = \mu_0 + \mu_1 \sin(\Omega t - Kx), \tag{4}$$

where Ω is the angular frequency of the ultrasonic wave, K is the wavenumber of the ultrasonic wave and x denotes the propagation direction of ultrasound. The Klein–Cook parameter divides light diffraction by an ultrasonic wave into three regimes [5]:

1. $Q \ll 1$ defines the so-called Raman and Nath region,
2. $Q \gg 1$ defines the Bragg regime,
3. $Q \approx 1$ defines the "intermediate range" or "transition region", which further covers light diffraction phenomena between the above two limiting cases.

The Raman–Nath equations system (1) has analytical solutions for either the Klein–Cook parameter $Q = 0$ or in case of $Q \gg 1$ and light incidence angle satisfying the so-called Bragg condition. In subsequent sections of the paper we will focus on the Raman–Nath and intermediate diffraction regions only. The phenomenon of Bragg diffraction of collinear photon pairs has been recently examined experimentally in a separate paper [3] where also relevant theoretical formulas are presented which are in a very good agreement with the obtained experimentally data. Therefore this diffraction regime will not be discussed here.

For $Q = 0$, the right-hand side of Eq. (1) equals zero and then the solutions to the set are expressed by Bessel functions. The amplitude of diffracted light in the n^{th} diffraction order appears to be described by n^{th} order Bessel function of the first kind:

$$\phi_n = (-1)^n J_n(v). \tag{5}$$

Whereas the intensity I_n of light in the n^{th} diffraction order can be expressed as:

$$I_n = \phi_n \cdot \phi_n = J_n^2(v), \tag{6}$$

where v is the Raman–Nath parameter (Eq. (2)). The last formula is valid for normal incidence of light on an ultrasonic wave ($\Theta = 0$). It should be noted (Fig. 1) that light in individual diffraction orders propagates at angles α_n, relative to the initial angle of light incidence on an ultrasonic wave, such that:

$$\sin\alpha_n = n\frac{\lambda}{\mu_0\Lambda}, \tag{7}$$

and the angular frequency, ω_n, of light in the n^{th} diffraction order is changed due to the Doppler effect:

$$\omega_n = \omega + n\Omega. \tag{8}$$

Eqs. (5) and (6), describing amplitude and intensity of light in separate diffraction orders, are valid only for small values of both the Raman–Nath and Klein–Cook parameters. And the region in which the amplitude and intensity of diffracted light are expressed by Bessel functions is called Raman–Nath region [5,6]. For bigger values of the Klein–Cook parameter Bessel-functions description is no longer valid and the amplitudes of diffracted light within individual diffraction orders have to be found as general solutions to the Raman–Nath system, Eq. (1). In this case they are expressed by complex functions and relevant diffracted light intensities are given as:

$$I_n = \phi_n \cdot \phi_n^*, \tag{9}$$

where ϕ_n^* denotes complex conjugate of ϕ_n.

Fig. 2 illustrates changes of diffracted light intensity distribution in the zero-th diffraction order, plotted as a function of the Raman–Nath parameter for five different values of the Klein–Cook parameter between 0 and 9. For the Klein–Cook parameter $Q = 5$, $Q = 7$, and $Q = 9$ the phenomenon of light rediffraction to the zero-th diffraction order can be observed [9], i.e. for some combinations of the Klein–Cook and Raman–Nath parameter values the entire energy of light returns to the zero-th diffraction order. When the Raman–Nath parameter value exceeds the boundary one at which the rediffraction phenomenon takes place for a given Klein–Cook parameter value, diffraction orders emerge again.

The quantum picture of light diffraction is usually formulated in single photons language. A light beam can be treated as a system of independent photons following the same probability distribution determined by a single photon wave function. The diffracting waves are solutions of the Maxwell equations (in our case the Raman–Nath system (1)) and can be interpreted as single photon wave functions [10]. And this is why the Deng's solution, based on Feynman's path integral approach, showing for single photons diffraction by an ultrasonic wave identical to that of Raman and Nath for a light wave is not a surprise.

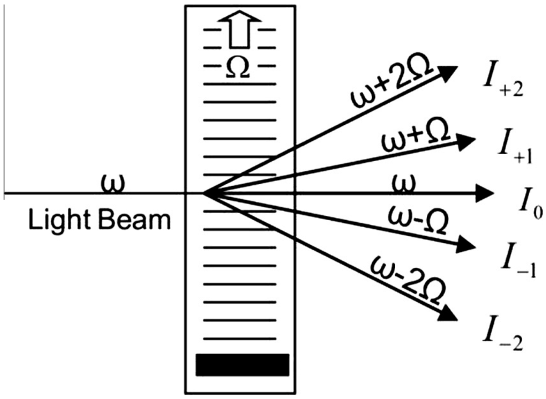

Fig. 1. Schematic representation of light diffraction by an ultrasonic wave in case of normal incidence of light on an ultrasonic wave.