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Nonlinear guided waves in plates: A numerical perspective

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ABSTRACT

Harmonic generation from non-cumulative fundamental symmetric (S_0) and antisymmetric (A_0) modes in plate is studied from a numerical standpoint. The contribution to harmonic generation from material nonlinearity is shown to be larger than that from geometric nonlinearity. Also, increasing the magnitude of the higher order elastic constants increases the amplitude of second harmonics. Second harmonic generation from non-phase-matched modes illustrates that group velocity matching is not a necessary condition for harmonic generation. Additionally, harmonic generation from primary mode is continuous and once generated, higher harmonics propagate independently. Lastly, the phenomenon of mode-interaction to generate sum and difference frequencies is demonstrated.

1. Introduction

Use of nonlinear ultrasound for characterizing microstructure of structural materials, especially metals, has been a topic of interest for several decades. Initial investigations by Breazeale and Thompson [1] and Hikata et al. [2] put forth elastic material nonlinearity and dislocations as predominant causes for nonlinear ultrasonic behavior i.e., higher harmonic generation. These results motivated the use of higher harmonic generation for investigating nonlinear behavior and hence material degradation in structures. Cantrell and Yost [3] investigated fatigued microstructures using nonlinear ultrasound. Cantrell [4] presented a comprehensive approach to relate the nonlinearity parameter (β) to the dislocation substructures in metals. For polycrystalline nickel, a monotonic increase in β with the fatigue cycles was predicted. The above theoretical/experimental investigations employed bulk waves (which travel in unbounded media) to characterize nonlinearity. On the other hand, guided waves (which travel in bounded structures) offer several advantages from an inspection standpoint in that long-range inspection can be carried out from a single location. Hence these are more amenable for structural health monitoring applications. Nonlinear guided waves combine the penetration power of guided waves with the early damage detection capabilities of nonlinear ultrasound. Hence they have emerged as an attractive alternative for detecting microstructural changes preceding macro-scale damage in the structures.

Deng [5,6] analyzed second harmonic generation from guided waves in plates. De Lima and Hamilton [7] presented a perturbation based approach to analyze second harmonic generation using the normal mode expansion technique [8] and arrived at two conditions necessary for cumulative second harmonic generation, namely phase matching and non-zero power flux. While bulk waves satisfy the above criterion for all frequencies of excitation, only specific guided wave modes satisfy them. These were identified [9,10] from the dispersion relations governing Rayleigh Lamb modes in the plate and experimental investigations [11–14] corroborated theoretical predictions. While the above investigations dealt with second harmonic generation, theoretical investigations were also carried out for predicting the nature of higher harmonic generation from guided waves in plates. Srivastava and Lanza di Scalea [15] concluded that cumulative even harmonics exist only as symmetric modes while odd harmonics can exist either as symmetric or antisymmetric modes. Chillara and Lissenden [16] presented a generalized theory to study the nonlinear interaction of guided wave modes. They concluded that the interaction of guided wave modes of the same nature generate symmetric modes while those of opposite nature generate antisymmetric modes. They also proposed a procedure to predict the nature of higher harmonics from the theory of mode interaction developed.

While a comprehensive theoretical framework is now available to study higher harmonic generation from guided waves in plates, some issues still need to be addressed. These stem from the following issues:

1. The theoretical analysis is carried out for time-harmonic (single-frequency continuous wave) excitations while the experiments employ transducers with finite band-width.

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2. Group velocity matching in addition to the phase velocity matching between the primary mode and other higher harmonics was assumed to be necessary for cumulative harmonic generation [17]. However, Deng et al. [18] presented an analysis and an experimental investigation confirming that it is not a necessary condition. In general, all the primary Rayleigh-Lamb modes capable of cumulative second harmonic generation in plates satisfy group velocity matching in addition to phase velocity matching [9]. However, with finite wave packets, it would be interesting to examine if the higher harmonic modes can propagate without group velocity matching as the non-zero-power-flux criterion which requires power flow from primary to higher harmonic modes may not be satisfied once the primary and higher modes separate due to difference in group velocity.
3. The analysis is generally carried out using a perturbation approach as no closed form solution is available.
4. The capability of the existing constitutive model to capture the effect of different kinds of nonlinearities has not been established.

In this article, we address some of the above issues using results from numerical simulations carried out in COMSOL Multiphysics 4.3, a commercial finite element software. Specifically, we consider the harmonic generation from the primary Symmetric (S_0) and Antisymmetric (A_0) modes in a plate. While neither of the above modes are exactly phase matched, they are deliberately chosen to highlight some of the aspects to be discussed later. As part of the study, we also comment on some aspects of the constitutive model like the contribution of material and geometric nonlinearities to higher harmonic generation.

The content of the article is organized as follows. Section 2 presents some continuum mechanics preliminaries. Section 3 then presents the results obtained from numerical simulations. Finally, conclusions are drawn in Section 4.

2. Preliminaries

In this section, we present some preliminaries intended to enhance understanding of the results to be presented. The discussion is brief so we refer the reader to [19] for more details.

We denote the deformation gradient by **F** and the Lagrangian strain by **E**. Also, the displacement gradient is denoted by **H** and the following relations exist between the above quantities.

$$\mathbf{F} = \mathbf{I} + \mathbf{H} \tag{1}$$

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^{\mathbf{T}}\mathbf{F} - \mathbf{I}) = \frac{1}{2}(\mathbf{H} + \mathbf{H}^{\mathbf{T}} + \mathbf{H}^{\mathbf{T}}\mathbf{H}) \tag{2}$$

We denote the linearized strain by $\mathbf{E}^{\mathbf{l}}$, which is related to the displacement gradient ($\mathbf{H}$) by

$$\mathbf{E}^{\mathbf{l}} = \frac{1}{2}(\mathbf{H} + \mathbf{H}^{\mathbf{T}}). \tag{3}$$

Note that the difference between the Langrangian strain and the linearized strain is that the second order term involving $\mathbf{H}^{\mathbf{T}}\mathbf{H}$ is dropped for the linearized strain. Including geometric nonlinearity means we consider Langrangian strain (full strain) as opposed to the linearized strain in the analysis.

A widely used constitutive model for studying higher harmonic generation was proposed by Landau and Lifshitz [20]. The corresponding strain energy function is given by

$$W(\mathbf{E}) = \frac{1}{2}\lambda(tr(\mathbf{E}))^2 + \mu tr(\mathbf{E}^2) + \frac{1}{3}C(tr(\mathbf{E}))^3 + Btr(\mathbf{E})tr(\mathbf{E}^2) + \frac{1}{3}A(tr(\mathbf{E}^3)). \tag{4}$$

where $\lambda,\ \mu$ are Lame's constants and $A,\ B,\ C$ are higher order elastic constants.

Another equivalent version of the above model is the Murnaghan model given by

$$W(\mathbf{E}) = \frac{1}{2}\lambda(tr(\mathbf{E}))^2 + \mu tr(\mathbf{E}^2) + \frac{1}{3}(l + 2m)(tr(\mathbf{E}))^3 - mtr(\mathbf{E}) \times ((tr(\mathbf{E}))^2 - tr(\mathbf{E^2})) + ndet(\mathbf{E}) \tag{5}$$

where $l,\ m$ and n are Murnaghan constants and $tr()$ and $det()$ denote trace and determinant of the tensor respectively. The relation between A, B, C and l, m, n are given in [21]; $l = B + C,\ m = \frac{1}{2}A + B$ and $n = A$.

The second Piola–Kirchhoff stress tensor is obtained using

$$\mathbf{T_{RR}} = \frac{\partial \mathbf{W}(\mathbf{E})}{\partial \mathbf{E}} \tag{6}$$

The first Piola Kirchhoff stress tensor **S** and Cauchy stress tensor **T** are given by

$$\mathbf{S} = \mathbf{F}\mathbf{T_{RR}} \tag{7}$$

and

$$\mathbf{T} = \mathbf{F}\mathbf{S}\mathbf{F}^{\mathbf{T}}. \tag{8}$$

While the first Piola–Kirchhoff stress tensor is used for a formulation in the reference configuration, Cauchy stress is used for a formulation in the current configuration.

3. Numerical simulations

In this section, we present results obtained from numerical simulations performed in COMSOL 4.3. All the simulations were carried out using the Murnaghan model (Eq. (5)) for Aluminum, the elastic constants of which are tabulated in Table 1. As mentioned earlier, fundamental symmetric and antisymmetric modes are used in the simulation. The schematic of the 2D model used for the simulation is shown in Fig. 1. The thickness of the plate is chosen to be 1 mm and the length of the plate is assumed to be 100 mm unless otherwise specified. Triangular elements are used to discretize the plate with a maximum element size of 0.1 mm and a minimum element size of 0.03 mm. The resulting mesh is then scaled by a factor of 1.5 (both along the length and the thickness) to obtain a finer discretization. A maximum time step of 0.01 μs is used for the simulation. Appropriate displacement boundary conditions are enforced on the left end of the plate to excite the intended modes. The x-component of the displacement is denoted by 'u' and the y-component is denoted by 'v' where the axes are shown in Fig. 1. The dispersion curves for the plate are shown in Fig. 2 and the primary modes used to study harmonic generation in the plate are indicated. None of the primary modes selected are phase matched to the secondary modes, so the second harmonics are not known as 'cumulative'. However, it is clear from both the theory [7] and the results that second harmonics are generated.

3.1. S_0 mode at 0.5 MHz

In this section, we present the results obtained for the S_0 mode at 0.5 MHz. This mode is almost phase matched to the second harmonic as the phase speed of the primary mode is 5.34 mm/μs

Table 1
Elastic constants in GPa used for simulation.

λ	μ	l	m	n
51	26	−250	−333	−350