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Characterization of a cylindrical rod by inversion of acoustic scattering data

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ABSTRACT

In this paper, a new approach is proposed for nondestructive characterization of immersed and embedded isotropic rod-shaped samples by inversion of acoustic scattering data. The normal mode expansion technique is used for modelling the scattered field and the compression incident and compression scattered waves are considered. Genetic algorithm is the inversion technique used for estimating the elastic wave velocities and density of the rods from their measured backscattered pressure spectrum. The inversion technique is capable of computing the parameter values that best fit a particular set of data. A perturbation study is conducted on the sensitivity of the resonance frequencies to changes in elastic properties and density of the rods. The numerical results indicate that proper selection of resonance frequencies leads to accurate measurement of elastic constants and density. The proposed approach showed very good convergence and the results obtained were found to agree very well with available data.

1. Introduction

Nondestructive evaluation (NDE) of materials is of high interest in many applications. These techniques are favourable because during the test, the specimen is not damaged or destroyed. What we are interested in is developing a technique for measuring the elastic constants and density of immersed and embedded rods. This could have applications in characterization of fibre-reinforced composites. The proposed technique is based on elastic wave scattering from cylindrical objects which include the cylindrical fibres used in fibre-reinforced composite materials. Scattering of acoustic waves from immersed cylindrical rods is a classical area that has been of interest to many scientists in various disciplines. The problem of acoustic wave scattering from a circular cylinder was first studied by Faran [1]. The principles of resonance scattering theory (RST) were developed by Flax et al. [2]. The resonance scattering theory explains the scattering of elastic waves from cylindrical objects and provides a mathematical model for this phenomenon [3–5]. RST shows that the backscattered amplitude spectrum can be decomposing into a background part and a resonance part [3,6–8]. Rhee and Park [8] proposed formalism for isolation of resonances in acoustic wave scattering from submerged fluid or elastic bodies. The resonances are obtained from the propagation of surface waves around the cylinder and are directly associated with elastic wave velocities and density of the material (and consequently with its elastic constants) [9–14]. Thus, these resonances can be used for characterization of various material properties such as elastic constants and density. A comprehensive study on resonance acoustic spectroscopy (RAS) was presented by Veksler [15]. A number of experimental methods have been developed and can be used for verification of the theoretical results [16–21]. The short-pulse method of isolation and identification of resonances (MIIR) [16,17,19–21] uses the Fast Fourier Transform (FFT) to obtain the experimental backscattered pressure field. Using these techniques, the form function, the resonance spectrum, and the mode number of individual resonances of a penetrable scatterer can all be measured [22].

Some work has also been carried out on characterization of cylindrical targets by inversion of acoustic scattering data [23–26]. Batard and Quentin [25] studied the variations of the resonance frequencies of an immersed cylinder due to changes in its wave velocities and density. They also developed an approximate method to solve the inverse problem and find the mechanical properties of an immersed elastic rod. Honarvar and Sinclair [26] measured the elastic properties of an immersed solid rod by matching the corresponding resonance frequencies of the experimental and theoretical form functions through an iterative computer algorithm. Karim et al. [27] used the simplex method for finding the properties of an adhesive layer in bonded plates. Yang and Yeh [28] used inversion methods for characterizing

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mechanical properties and geometry of a tube with axial and circumferential guided waves. Kari and Honarvar [29] proposed a new approach for nondestructive measurement of wave velocities in immersed rods by inversion of resonance scattering data. However, density determination of the immersed rods was not studied in their work.

Despite the large volume of work on acoustic wave scattering from immersed rods, studies on wave scattering from encased rods are relatively few. One of the major references on this topic is a book published in 1973 by Pao and Mow [30]. Flax and Uberall [31] studied the interfacial waves that arise when compression waves are incident on spherical inclusions inside a solid matrix. Beattie et al. [32] showed reasonable agreement between theory and experiment for plane waves normally incident on isotropic cylinders encased in an epoxy matrix. Sinclair and Addison [33] compared the experimental and theoretical diffraction spectra of a long SiC fiber in a titanium matrix. They showed that their technique has potential applications in characterization of the interfacial layer between the fibre and matrix. Scattering of plane acoustic waves from a transversely isotropic cylinder embedded in epoxy was investigated by Fan et al. [34]. They developed a mathematical model for the backscattered field, and the sensitivity of resonances to variations of the elastic constants of the cylinder was examined.

In this paper, we develop a new technique for measuring the elastic wave velocities and density of embedded and immersed rods. This could have applications in characterization of whiskers in fibre-reinforced composites. The technique is based on elastic wave scattering. The scattered fields from embedded and immersed cylinders is measured and fitted to their mathematically modelled counterparts through an inversion algorithm. The inversion algorithm used in this work is the genetic algorithm (GA). Genetic algorithms are a family of computational models inspired by the theory of evolution and so far these algorithms are generally the best and most robust kind of evolutionary algorithms [35,36]. The capability and accuracy of the proposed technique is demonstrated by a perturbation study conducted on an elastic cylinder. Moreover, experiments are conducted on immersed steel and aluminum rods and on the same steel rod embedded in an aluminum matrix and results are verified by comparison with available data.

2. Immersed cylindrical rod

The elastic properties of a single rod or a single fibre can be measured using acoustic wave scattering in an immersion setup. This method has been proposed in [26] and further enhanced in [29]. Since we will be using this technique for verification of our results, we first briefly review the mathematical model for acoustic wave scattering from immersed cylinders in this section. This also provides a good introduction to the mathematical model of wave scattering from embedded cylinders. We consider a cylindrical rod with outer radius a and density ρ_c submerged in a fluid with density ρ_f. An infinite plane acoustic wave of circular frequency ω is incident on the rod at an angle α_0, see Fig. 1.

A cylindrical coordinate system (r, θ, z) is chosen such that the z direction coincides with the axis of the cylinder. The solution method is the same as that used by Veksler [15]. The incident wave pressure (p_i) at an arbitrary point $M(r, \theta, z)$ is [15]:

$$p_i = P_0\sum_{n=0}^{\infty}\varepsilon_n i^n J_n(k_n r)\cos(n\theta)\exp[i(k_z z - \omega t)] \tag{1}$$

where $k_z = k \sin \alpha_0$, $k_n = k \cos \alpha_0$, $\varepsilon_n = 1$ for $n = 0$ and $\varepsilon_n = 2$ for $n > 0$, $k = \omega/c_f$ is the wave number associated with the sound speed in the fluid (c_f), P_0 is the incident pressure wave amplitude and J_n is the first kind Bessel function of order n. The scattered wave pressure (p_s) at an arbitrary point $M(r, \theta, z)$ is [26]:

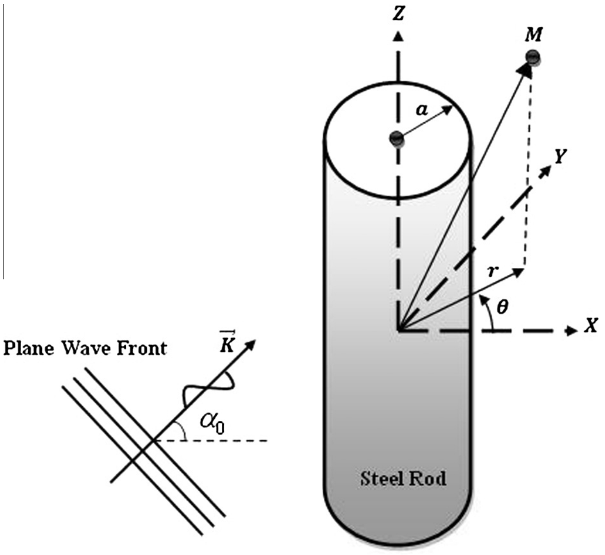

Fig. 1. Plane acoustic wave obliquely incident on a rod.

$$p_s = P_0\sum_{n=0}^{\infty}\varepsilon_n i_n A_n H_n^{(1)}(k_n r)\cos(n\theta)\exp[i(k_z Z - \omega t)] \tag{2}$$

where A_n are the unknown coefficients and $H_n^{(1)}$ is the first kind Hankel function of order n. The rod is considered to be isotropic and therefore:

$$c_L^2 = (\lambda + 2\mu)/\rho_c \quad \text{and} \quad c_T^2 = \mu/\rho_c \tag{3}$$

where λ and μ are Lame' constants, ρ_c is density, and c_L and c_T are the longitudinal and shear wave velocities, respectively. The non-dimensional backscattered pressure field, called form function is written as [26]:

$$f_\infty(\theta, ka) = \sum_{n=0}^{\infty} f_n(\theta, ka) \tag{4}$$

where [26],

$$f_n(\theta, ka) = \left(2/\sqrt{i\pi ka}\right)\varepsilon_n A_n \cos(n\theta) \tag{5}$$

The backscattered field is usually composed of a series of dips, which are due to resonances, superimposed on a smooth background [3,6–8,10–14]. To extract resonance frequencies from the form function, the background field should be known. By using the resonance scattering theory, the resonance information of the cylinder could be obtained by subtracting the proper background term (rigid cylinder) from the total scattered pressure field [8]:

$$f_n^{res}(\theta, ka) = \left(2/\sqrt{i\pi ka}\right)\varepsilon_n (A_n - A_n^r)\cos(n\theta) \tag{6}$$

where the coefficients of a rigid rod are obtained from Ref. [8]:

$$A_n^r = -J_n'(ka)H_n^{(1)'}(ka) \tag{7}$$

In Eq. (7), derivation is with respect to ka. By using Eqs. (6) and (7), the resonance part of the backscattered field can be completely separated from the background.

3. Embedded cylindrical rod

Let us consider the same rod as described in Section 2 which is now embedded in a solid elastic matrix. The density of the matrix