Ultrasonics 54 (2014) 1677–1684

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Wave propagation in layered piezoelectric rectangular bar: An extended orthogonal polynomial approach

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ARTICLE INFO

Article history:
Received 11 October 2013
Received in revised form 21 February 2014
Accepted 26 February 2014
Available online 12 March 2014

Keywords:
Guided waves
Rectangular bar
Piezoelectric materials
Multilayered structures
Orthogonal polynomials

ABSTRACT

Wave propagation in multilayered piezoelectric structures has received much attention in past forty years. But the research objects of previous research works are only for semi-infinite structures and one-dimensional structures, i.e., structures with a finite dimension in only one direction, such as horizontally infinite flat plates and axially infinite hollow cylinders. This paper proposes an extension of the orthogonal polynomial series approach to solve the wave propagation problem in a two-dimensional (2-D) piezoelectric structure, namely, a multilayered piezoelectric bar with a rectangular cross-section. Through numerical comparison with the available reference results for a purely elastic multilayered rectangular bar, the validity of the extended polynomial series approach is illustrated. The dispersion curves and electric potential distributions of various multilayered piezoelectric rectangular bars are calculated to reveal their wave propagation characteristics.

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1. Introduction

In the past four decades, wave propagation in piezoelectric structures has received considerable attention from engineering and scientific communities because of their applications in ultrasonic nondestructive evaluation and transducer design and optimization. For these piezoelectric devices, layered model consisting of piezoelectric and non-piezoelectric layers stacked in a certain sequence is common. Many solution methods have been used to investigate the wave propagation in multilayered piezoelectric structures, including analytical method [1,2] and various numerical method. The mostly used method is the transfer matrix method (TMM) [3–5] and the finite element method (FEM) [6].

Because the TMM and FEM suffer from numerical instability in some particular cases, some improvements have been developed, such as the recursive asymptotic stiffness matrix method [7,8], the surface impedance matrix method [9,10], the scattering-matrix method [11] and the reverberation-ray matrix method [12].

In 1972, the orthogonal polynomial approach has been developed to analyze linear acoustic waves in homogeneous semi-infinite wedges [13]. After that, this approach has been used to solve various wave propagation and vibration problems, including surface acoustic waves in layered semi-infinite piezoelectric structures [14,15], Lamb-like waves in multilayered piezoelectric plates, [16] multilayered piezoelectric curved structures [17,18] and multilayered magneto-electro-elastic plates [19].

So far, investigations on wave propagation in multilayered piezoelectric structures are only for semi-infinite structures and one-dimensional structures, i.e. structures having a finite dimension in only one direction, such as horizontally infinite flat plates and axially infinite hollow cylinders. But in practical applications, many piezoelectric elements have finite dimensions in two directions. One-dimensional models are thus not suitable for these structures. To the extent of the authors' knowledge, wave propagation in multilayered 2-D piezoelectric structure has not been reported. This paper proposes an extension of the orthogonal polynomial series approach to solve wave propagation problems in a 2-D piezoelectric structure, namely, multilayered piezoelectric bar with a rectangular cross-section. In particular, two cases are considered: the material stacking direction and the polarization direction are parallel and perpendicular, respectively. Traction-free

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http://dx.doi.org/10.1016/j.ultras.2014.02.023
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and open-circuit boundary conditions are assumed in this analysis. The wave dispersion curves and the electric potential profiles of various multilayered piezoelectric rectangular bars are presented and discussed.

2. Problem formulation and solution method

We consider a multilayered piezoelectric rectangular bar which is infinite in the wave propagation direction. Its width is d, the total height is $h = h_N$, and the stacking direction is in the z-direction, as shown in Fig. 1. Its polarization direction is in the z direction. The origin of the Cartesian coordinate system is located at a corner of the rectangular cross-section and the bar lies in the positive y–z-region, where the cross-section is defined by the region $0 \leqslant z \leqslant h$ and $0 \leqslant y \leqslant d$.

For the wave propagation problem considered in this paper, the body forces and electric charges are assumed to be zero. Thus, the elastodynamic equations for the rectangular bar are governed by

$$\begin{aligned}
&\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z}=\rho\frac{\partial^2 u_x}{\partial t^2},\\
&\frac{\partial T_{xy}}{\partial x}+\frac{\partial T_{yy}}{\partial y}+\frac{\partial T_{yz}}{\partial z}=\rho\frac{\partial^2 u_y}{\partial t^2},\\
&\frac{\partial T_{xz}}{\partial x}+\frac{\partial T_{yz}}{\partial y}+\frac{\partial T_{zz}}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2},\\
&\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=0,
\end{aligned} \tag{1}$$

where T_{ij}, u_i and D_i are the stress, elastic displacement and electric displacement components, respectively, and ρ is the density of the material.

The relationships between the generalized strain and generalized displacement components can be expressed as

$$\begin{aligned}
&\varepsilon_{xx}=\frac{\partial u_x}{\partial x},\quad \varepsilon_{yy}=\frac{\partial u_y}{\partial y},\quad \varepsilon_{zz}=\frac{\partial u_z}{\partial z},\quad \varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right),\\
&\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right),\quad \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\\
&E_x=-\frac{\partial \phi}{\partial x},\quad E_y=-\frac{\partial \phi}{\partial y},\quad E_z=-\frac{\partial \phi}{\partial z},
\end{aligned} \tag{2}$$

where ε_{ij}, E_i and ϕ are the strain components, electric field and electric potential, respectively.

We introduce the function $\mathrm{I}(y,z)$

$$\mathrm{I}(y,z)=\pi(y)\pi(z)=\begin{cases}1, & 0\leqslant y\leqslant d \quad \text{and} \quad 0\leqslant z\leqslant h\\ 0, & \text{elsewhere}\end{cases}, \tag{3}$$

where $\pi(y)$ and $\pi(z)$ are rectangular window functions $\pi(y)=\begin{cases}1, & 0\leqslant y\leqslant d\\ 0, & \text{elsewhere}\end{cases}$ and $\pi(z)=\begin{cases}1, & 0\leqslant z\leqslant h\\ 0, & \text{elsewhere}\end{cases}$. By introducing the function $\mathrm{I}(y,z)$, the traction-free and open-circuit boundary conditions ($T_{zz} = T_{xz} = T_{yz} = T_{yy} = T_{xy} = D_z = D_y = 0$ at the four boundaries) are automatically incorporated in the constitutive relations of the plate [14,15]

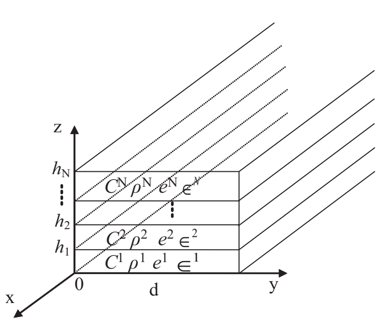

Fig. 1. Schematic diagram of a multilayered rectangular bar.

$$\begin{Bmatrix}T_{xx}\\T_{yy}\\T_{zz}\\T_{yz}\\T_{xz}\\T_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&C_{14}&C_{15}&C_{16}\\ &C_{22}&C_{23}&C_{24}&C_{25}&C_{26}\\ & &C_{33}&C_{34}&C_{35}&C_{36}\\ & & &C_{44}&C_{45}&C_{46}\\ &\text{symmetric}& & &C_{55}&C_{56}\\ & & & & &C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\mathrm{I}(y,z)\\ \varepsilon_{yy}\mathrm{I}(y,z)\\ \varepsilon_{zz}\mathrm{I}(y,z)\\ 2\varepsilon_{yz}\mathrm{I}(y,z)\\ 2\varepsilon_{xz}\mathrm{I}(y,z)\\ 2\varepsilon_{xy}\mathrm{I}(y,z)\end{Bmatrix} - \begin{bmatrix}e_{11}&e_{21}&e_{31}\\e_{12}&e_{22}&e_{32}\\e_{13}&e_{23}&e_{33}\\e_{14}&e_{24}&e_{34}\\e_{15}&e_{25}&e_{35}\\e_{16}&e_{26}&e_{36}\end{bmatrix}\begin{Bmatrix}E_x\mathrm{I}(y,z)\\E_y\mathrm{I}(y,z)\\E_z\mathrm{I}(y,z)\end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix}D_x\\D_y\\D_z\end{Bmatrix}=\begin{bmatrix}e_{11}&e_{12}&e_{13}&e_{14}&e_{15}&e_{16}\\e_{21}&e_{22}&e_{23}&e_{24}&e_{25}&e_{26}\\e_{31}&e_{32}&e_{33}&e_{34}&e_{35}&e_{36}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\mathrm{I}(y,z)\\ \varepsilon_{yy}\mathrm{I}(y,z)\\ \varepsilon_{zz}\mathrm{I}(y,z)\\ 2\varepsilon_{yz}\mathrm{I}(y,z)\\ 2\varepsilon_{xz}\mathrm{I}(y,z)\\ 2\varepsilon_{xy}\mathrm{I}(y,z)\end{Bmatrix} + \begin{bmatrix}\in_{11}&\in_{12}&\in_{13}\\ &\in_{22}&\in_{23}\\ \text{symmetric}& &\in_{33}\end{bmatrix}\begin{Bmatrix}E_x\mathrm{I}(y,z)\\E_y\mathrm{I}(y,z)\\E_z\mathrm{I}(y,z)\end{Bmatrix}, \tag{4b}$$

where C_{ij}, e_{ij} and $\boldsymbol{\epsilon}_{ij}$ are the elastic, piezoelectric and dielectric constants respectively.

The layered bar with a stacking direction being in the z-direction is denoted as z-directional layered bar. The layered bars considered in the following are all z-directional layered bars if not indicated specifically. The elastic constants of the layered bar are expressed as

$$C_{ij}(z)=\sum_{n=1}^{N}C_{ij}^{n}\pi_{h_{n-1},h_n}(z), \tag{5a}$$

where N is the number of the layers, C_{ij}^n is the elastic constant of the nth layer and $\pi_{h_{n-1},h_n}(z)=\begin{cases}1, & h_{n-1}\leqslant z\leqslant h_n\\ 0, & \text{elsewhere}\end{cases}$. Similarly, other material coefficients can be expressed as

$$e_{ij}(z)=\sum_{n=1}^{N}e_{ij}^{n}\pi_{h_{n-1},h_n}(z),\quad \varepsilon_{ij}(z)=\sum_{n=1}^{N}\varepsilon_{ij}^{n}\pi_{h_{n-1},h_n}(z),\quad \rho(z)=\sum_{n=1}^{N}\rho^{n}\pi_{h_{n-1},h_n}(z). \tag{5b}$$

For the layered bar with a stacking direction being in the y-direction, which is refered to as y-directional layered bar, the material constants are expressed as

$$\begin{aligned}
&C_{ij}(y)=\sum_{n=1}^{N}C_{ij}^{n}\pi_{h_{n-1},h_n}(y),\\
&e_{ij}(y)=\sum_{n=1}^{N}e_{ij}^{n}\pi_{h_{n-1},h_n}(y),\quad \varepsilon_{ij}(y)=\sum_{n=1}^{N}\varepsilon_{ij}^{n}\pi_{h_{n-1},h_n}(y),\quad \rho(y)=\sum_{n=1}^{N}\rho^{n}\pi_{h_{n-1},h_n}(y),
\end{aligned} \tag{6}$$

where $\pi_{h_{n-1},h_n}(y)=\begin{cases}1, & h_{n-1}\leqslant y\leqslant h_n\\ 0, & \text{elsewhere}\end{cases}$.

For plane time-harmonic waves propagating in the x-direction of a rectangular bar, we assume that the displacement components have the following form

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