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Guided wave propagation in uncertain elastic media

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ABSTRACT

In this paper, the authors present a numerical approach to study the guided elastic wave propagation in uncertain elastic media. Stochastic wave finite element method (S.W.F.E.M) formulation with consideration of spatial variability of material and geometrical properties is developed for probabilistic analysis of structures. The uncertain material properties are modelled as a set of random fields. The idea is to consider the random fields as a supplementary dimension of the problem through the spatial discretisation using the finite elements process. The stochastic forced response is formulated to study the stochastic dynamical behavior of the structure using the appropriate boundary conditions. In this work, a SWFE approach is employed in order to analyse the stochastic wave propagation and the numerical accuracy. The computational efficiency of the method is demonstrated by comparison with Monte Carlo simulations.

Contents

1. Introduction

Guided waves are still a subject of intensive research such as structural forms occur in several engineering areas. This research focuses on the study of guided wave properties and applications. Among the primary properties of guided waves to be given, we can mention the dispersion curve and the mode shapes. Dispersion curves give the velocity-frequency relationship for all the modes which may propagate in the studied structure. The guided wave mode shape gives the distribution of displacements in the normal section to the propagation axis. A wave finite element method (W.F.E.M) formulation provides an effective way to calculate the dispersion curves of complex guided structures and investigates there properties [1–4].

In the literature, however, most of founded numerical issues of wave propagation simulations are mainly limited to deterministic media. Numerical guided wave techniques characterisation in spatially homogeneous random media is investigated in this paper. The finite element method has been weakening in dealing with variation of structural uncertain parameters. In this context, the concept of a random field [5] should be studied. Due to the complexity of the structure, the perturbation of its parameters that arises from material and geometrical variability is often much higher than conventional standard structures. An accurate prediction of the uncertainty in performance of cylindrical pipes by introducing random variables or fields is thus desired. The uncertainties are often present in geometric properties, material characteristics and boundary conditions of the model. These variables are taken into account in models according to the both parametric [6,7] and non-parametric [8] approaches. Ichchou et al [6] considered the wave propagation features in random guided elastic media through the Stochastic Wave Finite Element Method (S.W.F.E.M) using a parametric probabilistic technique.

In this paper, a parametric approach for uncertainties treatments is considered and combined to the WFE technique. The

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method consists on considering the random fields as a supplementary dimension of the problem through the spatial discretisation using the finite elements process.

Generally, the stochastic characteristics of the random responses can be determined by studying the design parameter uncertainties which are often modelled by random variables. Hence, various techniques are suggested to solve such problems [9,10]. In the literature, there are many methods accounting for uncertainties. Monte-Carlo simulation, first-order reliability methods [11] and second-order reliability methods [12] are the frequently used methods of the reliability analysis. The primary objective of these methods is to compute the probability of failure associated with prescribed limit states. The second-moment analysis methods aim at characterising the second-order statistical moments, such as means and variance/covariance of response quantities [13]. To consider the spatial variability, random fields are usually used to represent the uncertain quantities [14]. By introducing a process of discretisation, random fields are expressed as vector of random variables. The statistics of interest of uncertain quantities are then achieved.

Due to the efficient representation of random fields, the spectral stochastic finite element method (SSFEM) proposed by Ghanem and Spanos [15] has been successfully applied to various kinds of stochastic problems. Guedri et al. [16] proposed a dynamic condensation method of stochastic models using a strategy combining the stochastic finite element method (SFEM) with the robust condensation model based on a discretisation technique of random fields that was established using the Karhunen–Loeve procedure [17]. A modal perturbation (MP) approach [18] leads an efficient calculation of the random eigenmodes and rapid synthesis of the random frequency response, thereby avoiding the bad conditioning of matrices around resonances. Liu [19] devised an analytical way enabling the stochastic finite element method (SFEM) to cope with uncertain parameter systems. This way is based on the mean centred second order perturbation method [20]. Applications of probability and perturbation concepts to standard finite element analysis can be numerically expensive, an efficient numerical solution procedure for the SFEM has been introduced by Liu [21]. Viktorovitch et al. [22] presented in his paper a complete and rigorous derivation of the well-known power flow equations by introducing two types of Gaussian random parameters in the description of the studied structures. The first one deals with the spatial position and the later with the location of the boundaries. An analytical method for dynamic analysis of systems with viscoelastic dampers has been developed by Hryniewicz [23]. The considered system is governed by the third order differential equation. The one-sided Green's function for deterministic and stochastic cases is derived in closed analytical form.

This contribution will extend mentioned works in order to provide a full numerical description of the stochastic problem simultaneously with some validations through the Monte Carlo simulations. The main contribution of this paper seems to be the stochastic forced response calculation. The structure is assimilated to a periodic waveguide composed of N identical substructures. The stochastic dynamical behavior is studied by using the appropriate boundary conditions and following an uncertainty introduced in the system parameters.

In this work, a SWFE approach is employed in order to analyse the stochastic dynamical behavior. Comparisons between numerical results and Monte Carlo simulations of the stochastic formulation are among the offered originalities of this work.

The paper contains 5 sections. In Section 2, the formulation of the stochastic wave finite element approach is presented through state vector representation. Section 3 provides the stochastic dynamical behavior formulation. Forced response is calculated following an uncertainty introduced in structure parameters and using the appropriate boundary conditions. Section 4 gives mainly numerical experiments. The formulation is general and the validations were given using the MC simulations. A conclusion together with a description of the work in progress is ultimately given.

2. Stochastic wave finite element method (SWFEM) formulation

In this subsection, a stochastic medium is considered. The same idea, such as finite element method, is used to develop the stochastic wave finite element approach based on the probabilistic tools. The idea is to consider the random fields as a supplementary dimension of the problem through the spatial discretisation using the finite elements process. The uncertainties are assumed to be mostly on the material properties. Such uncertainties are assumed to be spatially homogeneous. This guarantees the assumed periodicity will be respected in the non-deterministic case as in the deterministic situation. The pipe is assimilated to a periodic waveguide composed of N identical substructure (Fig. 1). The discretised model leads to give the stochastic dynamical equilibrium of any substructure k in this manner:

$$\widetilde{\mathbf{D}}\begin{pmatrix}\tilde{q}_L\\ \tilde{q}_R\end{pmatrix} = \begin{pmatrix}\widetilde{\mathbf{D}}_{LL} & \widetilde{\mathbf{D}}_{LR}\\ \widetilde{\mathbf{D}}_{RL} & \widetilde{\mathbf{D}}_{RR}\end{pmatrix}\begin{pmatrix}\tilde{q}_L\\ \tilde{q}_R\end{pmatrix} = \begin{pmatrix}\tilde{F}_L\\ \tilde{F}_R\end{pmatrix} \tag{1}$$

This equation presents the equation of motion condensed at the left and right boundaries of the substructure in order to respect the property of periodicity of the waveguide. $\widetilde{\mathbf{D}}$ represents the stochastic dynamic operator of the substructure, condensed on the dof's of the left and the right boundaries at the pulsation ω. $\tilde{q}$ and $\tilde{F}$ designate the stochastic displacements and loads, respectively.

Parametric approach considers the uncertain parameters (geometrical, material properties etc.) as random quantities. Specific discretisation approaches such as the stochastic finite element method (SFEM) which combines finite elements and probabilistic ways of thinking can be employed [15]. In this work, we use the parametric method to study the effect of each uncertain parameter separately in order to classify their severity. In fact, the correlations properties between these variables are not considered because, really, we can neglect the interactions between geometrical and material properties of the structure.

The random variables are modelled by Gaussian variables through a first order perturbation, mathematically: $\tilde{v} = \bar{v} + \sigma_v \varepsilon$ where $\tilde{v}$ is the random variable, $\bar{v}$ its mean, σ_v its standard deviation (perturbation) and ε is Gaussian variable centred and reduced. The polynomial chaos $(1, \varepsilon)$ is used as a supplementary dimension of the problem. Using the polynomial chaos projection of these variables, we can extract their means (deterministic quantity) and their standard deviations (dispersion). The first order development of stochastic variables is adopted, such that:

$$\begin{pmatrix}\overline{\mathbf{D}}_{\mathbf{LL}} + \boldsymbol{\sigma}_{\mathbf{D}_{\mathbf{LL}}}\varepsilon & \overline{\mathbf{D}}_{\mathbf{LR}} + \boldsymbol{\sigma}_{\mathbf{D}_{\mathbf{LR}}}\varepsilon\\ \overline{\mathbf{D}}_{\mathbf{RL}} + \boldsymbol{\sigma}_{\mathbf{D}_{\mathbf{RL}}}\varepsilon & \overline{\mathbf{D}}_{\mathbf{RR}} + \boldsymbol{\sigma}_{\mathbf{D}_{\mathbf{RR}}}\varepsilon\end{pmatrix}\begin{pmatrix}\bar{q}_L + \sigma_{q_L}\varepsilon\\ \bar{q}_R + \sigma_{q_R}\varepsilon\end{pmatrix} = \begin{pmatrix}\overline{F}_L + \sigma_{F_L}\varepsilon\\ \overline{F}_R + \sigma_{F_R}\varepsilon\end{pmatrix} \tag{2}$$

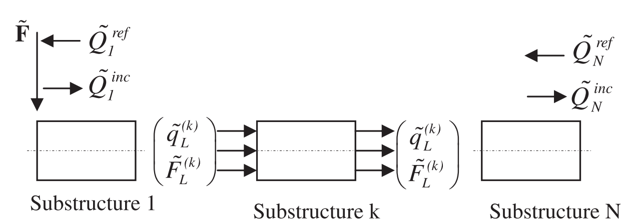

Fig. 1. An illustration of the periodic waveguide.