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Band gap structures in two-dimensional super porous phononic crystals

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ABSTRACT

As one kind of new linear cellular alloys (LCAs), Kagome honeycombs, which are constituted by triangular and hexagonal cells, attract great attention due to the excellent performance compared to the ordinary ones. Instead of mechanical investigation, the in-plane elastic wave dispersion in Kagome structures are analyzed in this paper aiming to the multi-functional application of the materials. Firstly, the band structures in the common two-dimensional (2D) porous phononic structures (triangular or hexagonal honeycombs) are discussed. Then, based on these results, the wave dispersion in Kagome honeycombs is given. Through the component cell porosity controlling, the effects of component cells on the whole responses of the structures are investigated. The intrinsic relation between the component cell porosity and the critical porosity of Kagome honeycombs is established. These results will provide an important guidance in the band structure design of super porous phononic crystals.

1. Introduction

Along with the increasing demands on the light-weight design, porous phononic crystals (PPCs), which have periodically distributed open or closed pores in bulk materials, have been widely used due to the potential application in the multifunctional fields, which include mechanical, thermal, and acoustic, etc. Recently, a new kind of porous phononic crystals – linear cellular alloy (LCA), which is constituted by two or more basic component cells, has attracted more and more attention due to its excellent performance over to the ordinary random cellular materials with the same porosity [1,2]. It is noticed that a lot of researches have been carried out to obtain the mechanical or thermal properties of these materials [3,4]. Whereas, as one kind of phononic crystals, another important characteristic, the existence of the absolute band gap (ABG) [5–7], which represents frequency regions where propagating elastic waves do not exist, also needs to be considered.

By now, some works about the wave dispersion in porous phononic crystals have been carried out. Liu et al. [8,9] investigated the influence of pore shapes and the lattice structure, as well as the lattice transformation, on the band structures in porous phononic crystals. Wang et al. [10] discussed the ABG structures in phononic crystals with cross-like holes. Yan and Zhang [11] analyzed the wave localization in two-dimensional porous phononic crystals with one-dimensional aperiodicity. Huang and Chen [12] analyzed the acoustic waves in two-dimensional phononic crystals with reticular geometric structures. Their results show that except for the bulk material and lattice structure, the topological properties of the pores have great influences on the ABG structures. Unfortunately, only the structures composed by one kind of basic pores are concentrated. Although some studies have been carried out to investigate the mechanical or thermal properties of LCAs, the band gap structures in these combined super porous phononic crystals, as well as the influence of the component cells on the total ABGs, have still not been clarified.

In order to provide some guidance in the further design of LCAs, the wave dispersion in one kind of widely used LCAs-the Kagome honeycomb, has been investigated. Firstly, by using FEM simulation, the ABGs in Kagome structures are given. Then, by fixing the lattice structure (square lattice) and comparing to the band structures in common PPCs (PPC constituted by one kind of component cells), the influence of the component cells on the whole ABGs in Kagome honeycombs are analyzed, which is helpful in the further application of LCAs in the acoustic design. At last, the conclusion is given.

2. Theory

Fig. 1 is a typical representation of the Kagome honeycomb with a super cell (denoted by I) periodically arranged in the 2D space, and the z-coordinate is set parallel to the axes of the pores, which are treated as vacuum. Then if the elastic waves propagate in the transverse plane ($x0y$ plane) with the displacement vectors independent of the z-coordinate, they can be decoupled into the mixed in-plane mode and the anti-plane shear mode. Accordingly, the in-plane wave equations are expressed in the frequency domain as:

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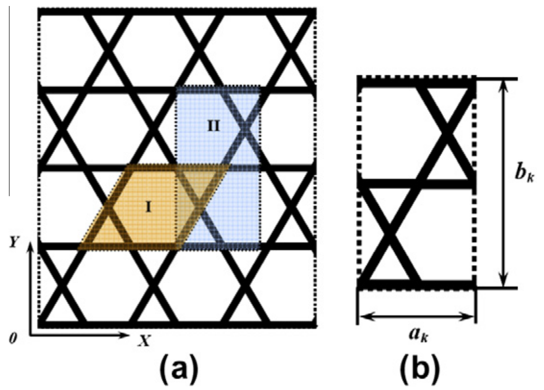

Fig. 1. (a) The cross section of Kagome honeycombs. (b) Super unit cell of Kagome honeycombs with a_k and b_k the lattice constants.

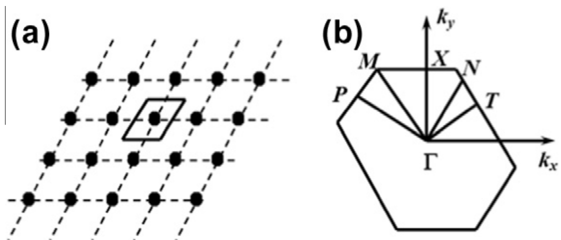

Fig. 2. (a) Parallelogram lattice. (b) The first Brillouin zone of parallelogram lattice.

$$-\rho(\mathbf{r})\omega^2 u_i = \nabla\cdot(\mu(\mathbf{r})\nabla u_i) + \nabla\cdot\left(\mu(\mathbf{r})\frac{\partial}{\partial x_i}\mathbf{u}\right) + \frac{\partial}{\partial x_i}(\lambda(\mathbf{r})\nabla\cdot\mathbf{u}),\quad (i = x,\ y), \tag{1}$$

In Eq. (1), $\mathbf{r} = (x, y)$ denotes the position vector, ω is the angular frequency, ρ is the mass density, λ and μ are the Lamé constant and shear modulus, $\mathbf{u} = (u_x, u_y)$ is the displacement vector in the transverse plane, and $\nabla = (\partial/\partial x, \partial/\partial y)$ is the 2D vector differential operator. According to Bloch theorem, the displacement field can be expressed as:

$$\mathbf{u}(\mathbf{r}) = e^{i(\mathbf{k}\cdot\mathbf{r})}\mathbf{u}_{\mathbf{k}}(\mathbf{r}), \tag{2}$$

where $\mathbf{k} = (k_x, k_y)$ is the wave vector limited to the first Brillouin zone of the reciprocal lattice and $\mathbf{u}_{\mathrm{k}}(\mathbf{r})$ is a periodical vector function with the same periodicity as the crystal lattice.

Considering the periodicity of the Kagome honeycomb, if super cell I (composed by a regular hexagon and two regular triangles) is chosen as the unit cell, the lattice structure is parallelogram (Fig. 2a) with the first Brillouin zone shown in Fig. 2b, that is, the wave vector should be valued along the boundary Γ-T-N-Γ-X-M-Γ-P-M to obtain the band structure, which increases the computing time. As a result, in the present manuscript, the super unit cell II (Fig. 1) is chosen as the computational domain. The parallelogram lattice is then changed to the rectangular lattice (Fig. 3a) with the first Brillouin zone Γ-X-M-Γ-N-M (Fig. 3b), which greatly improves the computational efficiency.

The Acoustic Module operating under the 2D plane strain Application Mode (ACPN) in COMSOL is applied to solve the governing equations. The discrete form of the eigenvalue equations in the unit cell can be written as:

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{U} = \mathbf{0}, \tag{3}$$

where $\mathbf{U}$ is the displacement at the nodes, $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices of the unit cell, respectively. The free boundary condition is imposed on the surface of the pore, and the Bloch boundary conditions on the two opposite boundaries of the unit cell, yielding:

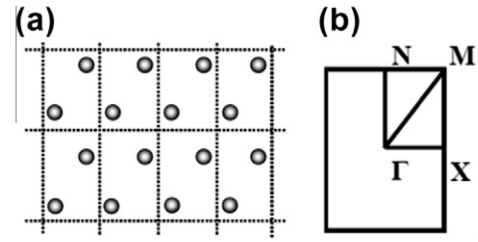

Fig. 3. (a) Rectangular lattice. (b) The first Brillouin zone of rectangular lattice.

$$\mathbf{U}(\mathbf{r}+\mathbf{a}) = e^{i(\mathbf{k}\cdot\mathbf{a})}\mathbf{U}(\mathbf{r}), \tag{4}$$

where $\mathbf{r}$ is located at the boundary nodes and $\mathbf{a}$ is the vector that generates the point lattice associated with the phononic crystals.

Through the maximum cell size and the change rate controlling, the representative cell, that is, cell II, is meshed by using the triangular Lagrange quadratic elements provided by COMSOL. Eigenfrequency analysis is chosen as the solver mode, and the direct SPOOLES is selected as the linear system solver. Moreover, the Hermitian transpose of the constrain matrix and parameter settings in symmetry direction in the advanced solver is required. The model built in COMSOL is saved as a MATLAB-compatible '.m' file. The file is programmed to let the wave vector $\mathbf{k}$ sweep the edges of the irreducible Brillouin zone, so that we can obtain the whole dispersion relations.

3. Numerical examples and discussion

As one kind of LCAs, the Kagome honeycomb could be seen as the overlap of a parallel cell with a regular hexagon and two regular triangles with the same side length (see, $a_k/2$, Figs. 1b and 4). The porosity, which is defined as the ratio between the pore area and the surface, is then calculated as:

$$f_K = f_T + f_H, \tag{5}$$

where f_T and f_H are the porosities of the triangular and hexagonal cells, respectively, which are given as:

$$f_H = 3(h-t)^2/4h^2, \tag{6}$$

$$f_T = (h-3t)^2/4h^2, \tag{7}$$

with $h = \sqrt{3}a_k/4$, and t the cell wall thickness.

Obviously, the porosity of the super cell is determined by the ones of the component cells. The porosity variation of the Kagome honeycomb could be obtained by changing the component cell wall thickness (hexagonal or triangular cells) equivalently (Fig. 5a, equal cell wall structure), or separately (only letting the cell wall thickness of one of the component cells be changed, Fig. 5b and c, that is, different cell wall structure), which results in the proportion variation of the component cells in the super ones.

In our discussion, in order to see clearly the influence of the component cells on the whole responses of the structures, the wave dispersion in common honeycombs composed by one kind of component cells (that is, regular triangular or hexagonal honeycombs) is investigated firstly. Based on these results, the variation of the band gap structures with respect to the porosity is discussed. It should be pointed out that the band gap structures rely on the matrix materials. Since topology is the main factor that we are interested, only one kind of basic materials is considered. The influ-